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Perverse cross-subsidization in the credit market*

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Abstract

We show how asymmetric information and borrowers' heterogeneity in wealth may produce equilibria in which, due to decreasing absolute risk aversion, hard working poor borrowers subsidize richer borrowers. In particular, a model of adverse selection and moral hazard in a competitive credit market is developed with private information on borrowers' wealth. Because of the ambiguous effect of decreasing risk aversion on the willingness to post collateral, both separating and pooling equilibria are possible in principle. Under separation the poor borrowers bear the cost of separation in terms of excessive risk taking. In a more likely pooling equilibrium poor hard-working borrowers subsidize richer ones.

Keywords: poverty, cross-subsidization, pooling and separating equilibria, unobservable wealth

JEL classification: D63, D8, H8

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1 Introduction

Does the credit market increase inequality of opportunity among individuals? How does the allocation of credit impact on distribution given one's wealth and effort? The "Voice of Small Business" survey[1] reported in 2009 that two thirds (61 per cent) of respondents said they have already or are considering switching banks because of their limited access to finance and lack of appropriate financial training and business support. A majority of small businesses are dissatisfied with their banks although they still rely heavily on them under necessity, and 1/3 of them reports specifically that credit should be allocated more fairly. All of these elements put on the alert about the role of lending constraints and wealth gaps in a market with strongly imperfect information.

Recent empirical evidence provides support for the importance of a *bank relationship* to small poor businesses in terms of both credit availability and credit terms such as loan interest rates and collateral. Berger and Udell (2002) for example suggest that the accumulation over time by the loan officer of "soft" information does play an important role to evaluate creditworthiness of poor business finance. Banks are also aware that success in entrepreneurial activity is dependent on some other alternative elements as human capital formation and it is not correlated with individuals' wealth (Cressy, 1996; a, b, c). On the other side wide empirical evidence supports the idea that capital market constraints prevent the poor from realizing entrepreneurial projects. In particular several studies evidence a substantial positive correlation between wealth and the probability of becoming an entrepreneur, once controlled for other variables, possibly suggesting that credit market imperfections impact negatively in particular on credit to the poor (see for example Black, de Meza and Jeffreys, 1996; Blanchflower and Oswald, 1998; Evans and Jovanovich, 1989).

With this evidence in mind, we explore the possibility that adverse selection produces perverse redistribution due to cross-subsidization in the wrong direction. The issue of cross-subsidization has been one of the most intensely discussed topics in the area, both in theory and practice (e.g., Black and de Meza, 1997; de Meza and Webb, 1999, 2000; de Meza, 2002), although more rarely it has been addressed from the equity point of view. Our contribution is grounded in the literature on inefficiencies in the credit market by de

[1] http://www.fsb.org.uk/News.aspx?loc=pressroom&rec=5742

Meza and Webb (1987), who point out the possibility that adverse selection in credit markets may lead to excessive entry into entrepreneurship due to cross subsidization, quite in contrast with the credit rationing phenomenon emphasized by Stiglitz and Weiss (1981). Inefficient investments may also occur notwithstanding collateral (Bester, 1985, 1987; Besanko and Thakor, 1987) serving as a signaling device (see Coco, 2000). Most related to our study however are the papers by Stiglitz and Weiss (1992), Coco (1999), de Meza and Webb (1999) and Gruner (2003). The first two papers demonstrate the impossibility of screening by collateral in the credit market with two classes of borrowers differing for their risk attitude. Risk preferences and project quality interact through moral hazard in conflicting ways, so that collateral is not any more a meaningful signal of project quality. In Stiglitz and Weiss (1992) in particular, differences in risk attitude arise due to decreasing risk aversion, an idea we will exploit in this paper as well. De Meza and Webb (1999) instead demonstrate that, even when rationing arises in a pooling equilibrium and participation to the credit market is wealth-dependent[2], there may still be overlending due to cross subsidization. Another important contribution, due to Gruner (2003), shows conditions under which intrinsically unproductive rich borrowers crowd out productive poor ones, suggesting that an ex-ante complete redistribution of endowments may lead to an improvement in aggregate welfare by increasing the risk-free interest rate. Finally a recent contribution by Coco and Pignataro (2010) shows that when entrepreneurs' heterogeneity concerns both wealth and aversion to effort, and this last variable is unobservable, wider cross subsidization in high wealth classes may lead to a violation of the equality of opportunity principle.

Most of these contributions duly focus their attention on the efficiency properties of equilibria. When moral hazard and adverse selection exist simultaneously in the credit market as in our case, then it is of interest to determine how these two imperfections interact with differing wealth endowments to determine the distributional impact of credit allocation. Our model allows for unobservable wealth with decreasing absolute risk aversion, to show how poor individuals ending up as hard-working agents, are, in a perverted logic, those who systematically cross subsidize rich individuals in equilibrium. Asymmetric information on wealth is not the customary assumption

[2] That is when the model displays the empirically well-established fact that wealthier individuals are more likely to participate in the credit market.

and it may appear strange at first sight. However there seem to be both the possibility and the reasons to conceal one's wealth from a bank during the borrowing process, particularly, as in this case, when, due to decreasing risk aversion, more wealth is a bad signal.

Specifically, risk aversion and effort choice interact in the determination of the willingness to post collateral and therefore determine when the equilibrium is pooling versus separating. Risk aversion impacts the willingness to post collateral both directly and through effort choice in opposite ways. When the moral hazard channel is more important a separating equilibrium is in principle possible, using collateral as screening device. The net cost of separation is shouldered by the poor individuals in the form of excess risk taking. When the direct effect of risk aversion prevails (e.g. the single crossing property is violated) then pooling is the only possible adverse-selection equilibrium. This case is the most interesting one to investigate. Cross-subsidization naturally occurs in the pooling equilibrium with the poor hard-working borrowers subsidizing rich borrowers. The rich are therefore charged a low rate of interest (relative to their risk) while the poor borrowers are charged too high an interest rate. This mechanism is empirically confirmed by Cressy (2000) who suggests, on the basis of evidence, that wealthy individuals due to decreasing absolute risk aversion, have a higher inclination to take on risky assets.

The structure of the paper is as follows. Section 2 introduces the baseline model while in Section 3 we start with characterizing the agents' preference map. Section 4 investigates the potential equilibria in the market. Conclusions follow in section 5.

2 The model

2.1 The Projects

We consider a one-period closed economy in a competitive credit market where each project requires an amount of capital K. It yields a gross return Y with probability of success $p(e)$ or zero revenue in case of failure with probability $1-p(e)$, where $e \in [0, \bar{e}]$ is the amount of effort and the measure of its utility cost. Returns to effort are positive and diminishing as usual, i.e. $p'(e) > 0$ and $p''(e) < 0$. In more general terms, higher levels of effort e result in a project whose returns first-order stochastically dominates (FOSD) the return of projects with lower level of effort.

2.2 Entrepreneurs

There is a finite number of would-be borrowers each endowed with a project described above. Borrowers are risk averse. Particularly, the individual's expected utility is a concave increasing function that exhibits decreasing absolute risk aversion, i.e., $d(-U''(w)/U'(w))/dw < 0$ and $U(w = 0) = -\infty$. Let $X = (1+r)K$ be the total repayment where r is the interest rate required by the bank for the amount of collateral c. Further, each agent has a different amount of wealth w_i $\forall i \in [R, P]$, respectively, poor and rich, which are both insufficient to full collateralization, $w_i < (1+r)K$. The borrower's wealth w_i and her actual effort choice e_i are assumed to be private information. In the two-state case concerning our analysis, the expected utility of an individual i is:

$$U_i = p(e_i)U(Y - X + w_i) + (1 - p(e_i))U(w_i - c) - e_i \tag{1}$$

2.3 Banks

A fixed amount of capital K is financed by risk-neutral lenders. Bertrand competition in credit market implies that in equilibrium banks earn zero expected profit and so eq. (2) below is equal to zero. Under ex-ante asymmetric information, lenders know the wealth distribution of borrowers (a fraction λ of these entrepreneurs belongs to rich types while $(1-\lambda)$ are poor ones), but cannot observe the particular borrower's wealth when a loan application is made. We assume zero risk-free interest rate and an infinitely elastic supply of funds in the deposit market. Under these conditions the standard optimal form of finance would be equity, but we assume that ex-post returns are unverifiable, and therefore the only viable form of finance is debt (see de Meza and Webb, 2000). For a single borrower, the representative bank's profit in a competitive market is:

$$\pi_i = p(e_i)X + (1 - p(e_i))c - K \tag{2}$$

3 Agents' preference map

If a lender can observe a borrower's level of effort and can write for example an effort contingent contract, then, there is no moral hazard and a first-best outcomes will potentially emerge. Instead, when effort is unobservable, the bank must infer $e^*(w, X, c)$, the borrower's optimal level of effort as a function of wealth and repayment of the project.

Using eq. (1), the first order condition for the borrower's optimal choice of effort $e^*(w, X, c)$ is given by:

$$\frac{\partial U_i}{\partial e_i} = p'(e_i)U(Y - X + w_i) - p'(e_i)U(w_i - c) = 1 \tag{3}$$

Eq. (3) shows that the borrower supplies effort until the expected value of marginal effort equals the marginal cost of effort. Rearranging eq. (3), the optimal choice of effort $e_i^*(w, Y, X, c)$ is described by:

$$p'(e_i^*) = \frac{1}{U(Y - X + w_i) - U(w_i - c)} \tag{4}$$

From straightforward comparative statics it follows that $\frac{de^*}{dY} > 0$; $\frac{de^*}{dc} > 0$; $\frac{de^*}{dX} < 0$, as is customary in moral hazard models. A higher repayment negatively affects the borrower's return in case of success, but not in the case of failure, thus reducing incentives to apply more effort. On the other side, a higher amount of collateral reflects higher penalty in case of failure providing incentives to put in effort.

With a similar argument, one can show that there exists a negative relation between effort and wealth, i.e., the marginal effort is lower, the higher the wealth of individuals:

$$\frac{de^*}{dw} < 0 \tag{5}$$

Proof. See the Appendix

To explore the type of equilibria that may arise in this context, it is now useful a diagrammatic representation of the equilibrium as follows. Using (1) and from the Envelope Theorem, we know that the slope of an indifference curve of a borrower in the $(X, c)-$ space is

$$\frac{dX}{dc} < 0 \tag{6}$$

Proof. See the Appendix

The crucial element to establish the possibility of separating equilibria is the observation of the slope of the indifference curves in the (X, c) space in relation to the wealth of borrowers. In this respect we may separate the effect of risk preferences from the impact of moral hazard. We can therefore rewrite the slope of the indifference curve in (6) as:

$$\frac{dX}{dc} = M(w)P(w)$$

where $M(w) = -\frac{(1-p(e_i))}{p(e_i)}$ while $P(w) = \frac{U'(W^F)}{U'(W^S)}$. The curvature of the indifference curve with respect to change in wealth is then:

$$\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) = M(w)R'(w) + M'(w)R(w) \gtrless 0 \tag{7}$$

Proof. See the Appendix

Here, $M(w)R'(w)$ captures the risk preference effect while $M'(w)R(w)$ explains the impact of moral hazard. Not surprisingly (7) has an ambiguous sign. On one side, the effect of (decreasing) risk aversion makes the indifference curve flatter as wealth increases. On the other side the negative impact of moral hazard makes it steeper. Indeed, for a given project choice, due to decreasing absolute risk aversion, rich individuals require a smaller reduction in the repayment rate to compensate for an increase in collateral[3]. Whenever the impact of moral hazard prevails as in eq. (8), rich individuals put such a low level of effort, and their probability of success diminishes by so much that their trade-off between collateral and interest rate becomes worse than poor people's one, notwithstanding their lower risk aversion:

[3] This is the case naturally proposed by Stiglitz and Weiss (1992).

$$\frac{\partial e}{\partial w} > p(e_i)(1 - p(e_i))(A(W^S) - A(W^F)) \tag{8}$$

Proof. See the Appendix

Note that this ambiguity in general means that the single crossing property of indifference curves which is a necessary condition to ensure the possibility of separation does not hold. Let us now consider the slope of the isoprofit curve for a bank lending to the borrower of class i only:

$$\frac{dX}{dc}|_{\bar{\pi}_i} = -\frac{(1 - p(e_i)) + (dp(e_i)/dc)\,(X - c)}{p(e_i) + (dp(e_i)/dX)\,(X - c)} \tag{9}$$

where $\bar{\pi}_i$ is the bank's expected profit on the borrower of class i. Since $dp(e_i)/dX$ is negative, (9) could in principle be positive. Note that this becomes more likely for high values of X and correspondingly low values of $p(e_i)$ and c, see Coco (1999). We may immediately note that, by construction, under this information structure, individuals with a larger wealth (higher risk from the point of view of banks) may prefer contracts that are actuarially fair for poor individuals, e.g., on line O_1 in figure 1, due to decreasing risk aversion. This makes separation in pure strategies impossible when risk aversion prevails.

4 Perverse cross-subsidization

Under hidden information lenders just know the distribution of classes of wealth and the shares of the subgroups of the population. Figure 1 describes the main elements of the model and the potential equilibrium which allows the bank to break even. The indifference curves for the rich and poor individuals are labeled respectively I_R and I_P, while the isoprofit lines for rich, pooling and poor individuals are defined as O_P, $O_{POOLING}$, O_R.

From eq. (7) due to decreasing absolute risk aversion, individuals with lower wealth (and thus with greater risk aversion) may have a steeper or flatter indifference curves in different portions of the $(X, c)-$space, while considering eq. (5), they are also the 'hard-working' agents at any given contract. We will discuss in this section two polar 'well behaved' cases arising when (7) is always positive or always negative and the single crossing property holds one way or the other.

In figure 1, indifference curves are drawn according to the hypothesis that the impact of risk aversion exceeds the effect of moral hazard, hence $\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) > 0$. In this case, because of decreasing absolute risk aversion and its negative impact on effort, rich individuals display a relative preference for posting more collateral compared to the poor ones at any point in the space (X, c) notwithstanding their lower success probabilities.

Each bank offers a contract that maximizes their profit, while agents choose the best contract among the different offered alternatives. An equilibrium is a set of contracts offered by banks consistently with each other in a competitive market. The Nash concept in pure strategies is applied, implying that in an equilibrium, no bank is able to offer another contract on which it obtains an expected return (e.g. profit) higher than the equilibrium one, given each entrepreneurs' types choice of contract[4]. Under these conditions, separation is impossible as there exists no contract on O_P that can attract low wealth/low risk borrowers while deterring the wealthier ones. The relative slope of the indifference curves of the two types is inconsistent with the use of collateral as a signal (as in Coco, 1999). Therefore the equilibrium outcome is for each bank to offer a pooling contract, C_{POOL}, that results in a competitive return when chosen by both types of borrowers and maximizes on $O_{POOLING}$, the utility of low wealth/low risk borrowers. To convince yourself note that any other contract on the pooling zero profit line, $O_{POOLING}$, can be beaten by an additional contract that steals away the good-risk borrowers and leaves it with only the worst borrowers and negative profits. Separation as we said is ruled out by the borrowers' preference pattern. Cross-subsidization naturally occurs in the adverse selection pooling equilibrium in pure strategies. Note moreover that contrary to other settings (Coco, 1999), here, the pooling contract does not need to be a zero collateral one (although it could). Notwithstanding the fact that borrowers are risk averse and the bank risk neutral, posting collateral (and a lower interest rate) increases the surplus from the project. The additional surplus accrues entirely to borrowers and may compensate, especially for low values of col-

[4]That is zero profit given Bertrand Competition. See Rothshild and Stiglitz (1976) for the definition of equilibrium in pure strategy.

lateral where the incentive effect is supposed to be larger and risk aversion lower, the additional risk.

Let's turn now to the second case. When the moral hazard effect prevails with respect to the impact of risk aversion, the slope of the indifference curve of rich borrowers is steeper than the indifference curve of poor ones. In this case asymmetric information may in principle be overcome by the use of collateral as a sorting device. Borrowers can be screened by the offer of appropriate pair of contracts.

In this case a menu of contracts $C_i = (X_i, c_i)\ \forall i \in [R, P]$ will be offered in equilibrium by the representative lender in order to maximize its expected profit:

$$\begin{aligned} \pi \quad = \quad & \lambda[p(e_R)X + (1 - p(e_R))c_R - K] + \\ & (1 - \lambda)[p(e_R)X + (1 - p(e_R))c_R - K] \end{aligned} \tag{10}$$

According to the revelation principle, the bank needs to restrict contract profiles ensuring that, each entrepreneur would get the contract designed for her type (incentive compatibility) and, that the two agent types would be willing to accept their respective contracts under individual rationality:

$$\begin{aligned} p(e_R)U(Y - X_R + w_R) + (1 - p(e_R))U(w_R - c_R) - e_R \geq \\ p(e_R)U(Y - X_P + w_R) + (1 - p(e_R))U(w_R - c_P) - e_R \end{aligned} \tag{11a}$$

$$\begin{aligned} p(e_P)U(Y - X_P + w_P) + (1 - p(e_P))U(w_P - c_P) - e_P \geq \\ p(e_P)U(Y - X_R + w_P) + (1 - p(e_P))U(w_P - c_R) - e_P \end{aligned} \tag{11b}$$

$$p(e_i)U(Y - X_i + w_i) + (1 - p(e_i))U(w_i - c_i) - e_i \geq 0 \qquad \forall i = R, P \tag{12}$$

Of course competition results in zero profits at each contract and the chosen contract for poor hard-working agents is the one that minimizes their collateral, keeping incentive compatibility.

As described in figure 2, the two contracts proposed are indifferent for the rich individuals, while poor ones strictly prefer C_P, given their higher risk aversion and lower risk. Considering that each contract is on the bank's break-even line, under the further assumption that $U_P(C_P) > U_P(C_{POOL})$[5]

[5]Note that under the opposite assumption the indifference curve of poor borrowers passing through C_P intersects the vertical axis at a point below the break-even pooling profit line. In this case the equilibrium does not exist because it is always possible to design preferred pooling contracts for poor borrowers. A pooling contract however is not equilibrium either (see Rothshild and Stiglitz, 1976).

with the pooling contract , C_{POOL}, at zero collateral $c = 0$, thus, no other contract (hypothetically labeled C_N) can attract any bundle of the two entrepreneurs and make a positive expected profit. Note that this requires a zero collateral contract just for rich borrowers. In this case in which moral hazard prevails, rich individuals put such a low level of effort that their probability of success is much lower and they prefer to pay a higher repayment in case of success and no collateral for a probable failure. The collateral posted in C_P is the minimum amount required to avoid cross-subsidization among classes of wealth. Of course separation occurs at a cost, a deadweight loss which, in this case, is borne by the poor good-quality borrowers. An inefficient amount of collateral is posted in this case to signal their quality type, and a cost in term of inefficient risk allocation arises, entirely shouldered by poor good-risk types.

Outside these two cases, the indifference curves do not respect the single crossing property and as a general rule separation is impossible. Pooling equilibria are possible but only depending on the relative slope of the indifference curves at the candidate zero-profit contract. In case of pooling, cross subsidization is bound to occur just as in the first case described above (Figure 1). Hence outside the possible but unlikely case where conditions in eq. (11 a, b) and (12) happen in each point on the $(X, c)-$ space, a pooling equilibrium is the only possible equilibrium.

5 Concluding remarks

The argument of this work is the possibility that credit market equilibria entails adverse distributive effects, in particular through perverse cross-subsidization from poor to rich agents.

To this aim we build a model of the credit market where otherwise equal borrowers, choose the effort to put in a project based on the only heterogeneous dimension, their unobservable wealth. Because of decreasing risk aversion, moral hazard impacts more wealthier entrepreneurs. Willingness to post collateral here is ambiguously correlated with wealth. On one side wealthier entrepreneurs are less risk adverse and therefore, other things equal more willing to post collateral. On the other they are riskier because of moral hazard and therefore less willing. When the risk aversion effect is higher than negative moral hazard one, it becomes impossible to separate borrowers in equilibrium because the potential signal (collateral) is useless. In this case cross subsidization among unobservable classes of wealth naturally occurs in a pooling equilibrium. What strikes about this kind of equilibrium is the direction of the cross-subsidization. Poor hard-working borrowers subsidize rich 'lazy' borrowers. As always, cross-subsidization implies a welfare loss due to lower effort of the good risk types because of higher than necessary interest rates. But the overall welfare consequences of cross subsidization relative to a separating equilibrium are unclear because on the other hand bad risk types are benefited by lower interest rates.

Whenever the impact of moral hazard prevails in the whole contract space, the only possible equilibrium is a separating one through the use of the screening device. Collateral requirement in this case, is a net cost paid by the poor individuals and, as a consequence, their net welfare will be again lower than under full information.

In all possible cases poor borrowers lose out from asymmetric information and the presence of rich borrowers. Asymmetric information worsens the distribution of resources in society. Up to now, the belief that credit market imperfections could increase inequality was widespread, but in the received wisdom this effect came through credit availability. While we believe this channel to be relevant, we discover here a different reason to believe that there are adverse distributional effects from credit market imperfections.

These results imply that State programs promoting entrepreneurial creation need to be refined. In the last decade, a conjecture about the potential welfare costs of exclusion has led to widespread government intervention in the banking sector, particularly, of low income countries (Burgess, Pande and Wong; 2005). Examples of such interventions range from interest rate ceilings on lending to the small poor entrepreneurs to a mix of taxes (for the infra-marginals) and subsidies (for the rationed marginals). Whether such interventions actually improve the access of the more efficient poor to banks, help alleviate poverty and wealth gaps or both, remains widely debated. Our research particularly confirms that such programs concentrated on specific target groups, may help unwind undesirable cross subsidization between classes of borrowers. An appropriate design of programs requires that they should be focused on personal characteristics of potential entrepreneurs in the opportunity egalitarian perspectives.

A useful avenue for further research in this area is the exploration of the interaction between subsidization and exclusion (or alternatively with participation and entry). In our setting the equilibrium entails that all entrepreneurs are served. However in a richer setting, allowing for rationing for example, poorer borrowers may be crowed out by richer entrepreneurs. Or alternatively inefficient entry of rich individuals can be triggered by subsidization leading to an even worse outcome.

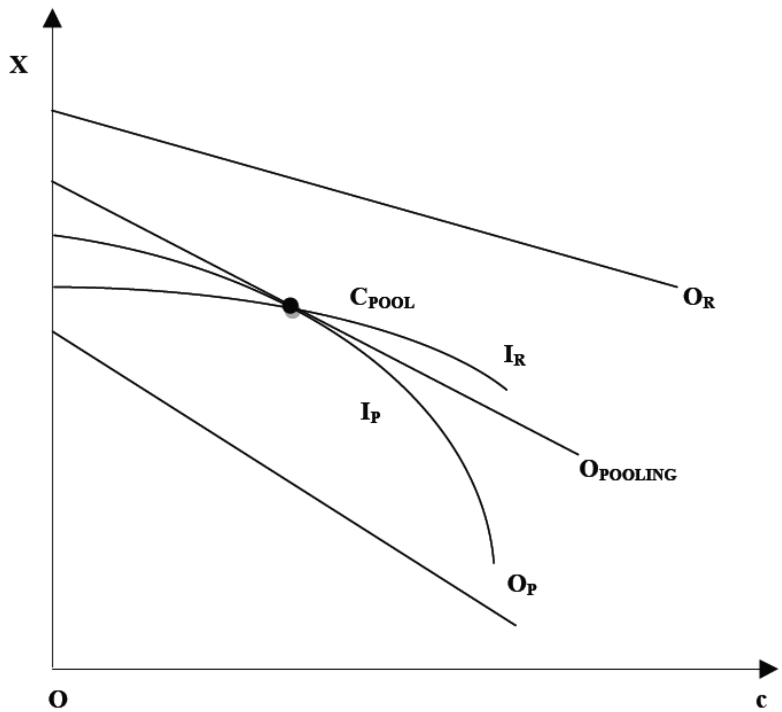

FIGURE 1: POOLING EQUILIBRIUM

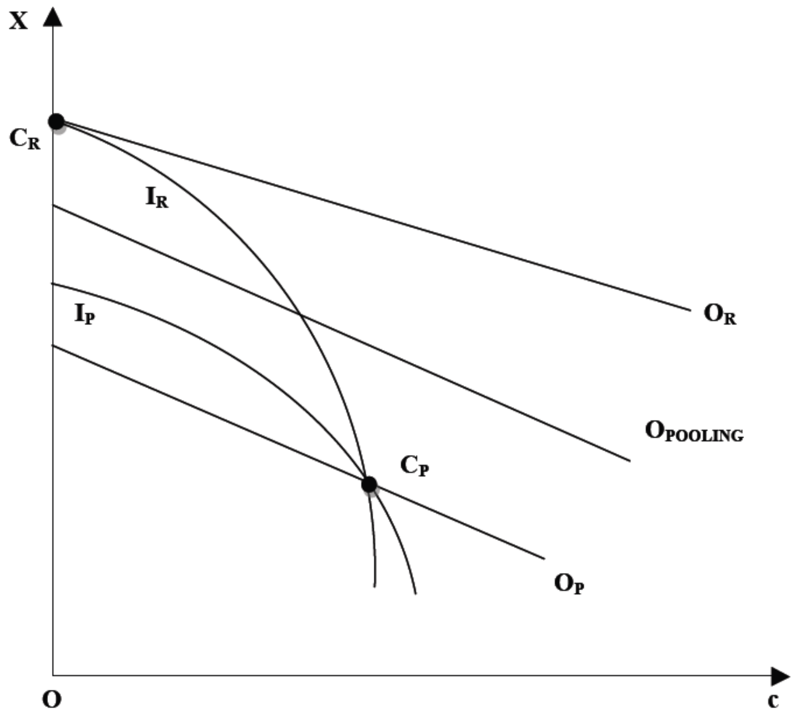

FIGURE 2: SEPARATING EQUILIBRIUM

6 The Appendix

A) Proof of eq. (5):

Starting by eq. (3):

$$\frac{\partial U_i}{\partial e_i} = p'(e_i)\left(U(W^S) - U(W^F)\right) - 1$$

By the Implicit Function theorem and due to decreasing absolute risk aversion, we simply observe that:

$$\left[p''(e_i)\left(U(W^S) - U(W^F)\right)\right] de + \left[p'(e_i)\left(U'(W^S) - U'(W^F)\right)\right] dw = 0$$

$$\left[p''(e_i)\left(U(W^S) - U(W^F)\right)\right] de = -\left[p'(e_i)\left(U'(W^S) - U'(W^F)\right)\right] dw$$

which implies that:

$$\frac{de}{dw} = -\frac{p'(e_i)\left(U'(W^S) - U'(W^F)\right)}{p''(e_i)\left(U(W^S) - U(W^F)\right)} < 0$$

B) Proof of eq. (6):

Starting by eq. (1):

$$U_i = p(e_i)U(Y - X + w_i) + (1 - p(e_i))U(w_i - c) - e_i$$

Let us assume that $W^S = Y - X + w_i$ and $W^F = w_i - c$, we can simply rewrite that:

$$U_i = p(e_i)U(W^S) + (1 - p(e_i))U(W^F) - e_i$$

By envelope theorem and differentiating with respect to X and c, it follows that:

$$\left[-p(e_i)U'(W^S)\right] dX - \left[(1 - p(e_i))U'(W^F)\right] dC = 0$$

$$\left[-p(e_i)U'(W^S)\right] dX = \left[(1 - p(e_i))U'(W^F)\right] dC$$

which implies that:

$$\frac{dX}{dc} = -\frac{(1-p(e_i))U'(W^F)}{p(e_i)U'(W^S)} = s < 0$$

C) Second order condition $\left(\frac{d^2X}{dc^2}\right)$

Let us define as s the slope of the indifference curve. The curvature of the indifference curve can be studied after differentiating eq. (6) with respect to c:

$$\frac{d^2X}{dc^2} = -\begin{array}{c}\{\frac{\left[-(1-p(e_i))U''(W^F)-p'(e_i)\frac{\partial e}{\partial c}U'(W^F)\right]\left(p(e_i)U'(W^S)\right)}{[p(e_i)U'(W^S)]^2} \\ -\frac{\left[(1-p(e_i))U'(W^F)\right]\left[-p(e_i)U''(W^S)\frac{\partial X}{\partial c}+p'(e_i)\frac{\partial e}{\partial c}U'(W^S)\right]}{[p(e_i)U'(W^S)]^2}\}\end{array} =$$

$$= -\begin{array}{c}\{-\frac{(1-p(e_i))U''(W^F)}{[p(e_i)U'(W^S)]} - \frac{p'(e_i)\frac{\partial e}{\partial c}U'(W^F)}{[p(e_i)U'(W^S)]} + \frac{p(e)(1-p(e))U'(W^F)U''(W^S)\frac{\partial X}{\partial c}}{[p(e_i)U'(W^S)]^2} \\ -\frac{p'(e_i)\frac{\partial e}{\partial c}U'(W^S)(1-p(e_i))U'(W^F)}{[p(e_i)U'(W^S)]^2}\}\end{array} =$$

$$= \begin{array}{c}\{\frac{(1-p(e_i))U''(W^F)}{p(e_i)U'(W^S)} - s\frac{(1-p(e_i))U''(W^S)U'(W^F)}{p(e_i)(U'(W^S))^2} + \frac{p'(e_i)\frac{\partial e}{\partial c}U'(W^F)}{p(e_i)U'(W^S)} \\ +\frac{p'(e_i)\frac{\partial e}{\partial c}(1-p(e_i))U'(W^F)}{p(e_i)^2U'(W^S)}\}\end{array} =$$

$$= \begin{array}{c}\{\frac{(1-p(e_i))}{p(e_i)}\frac{U'(W^F)}{U'(W^S)}\left(\frac{U''(W^F)}{U'(W^F)} - s\frac{U''(W^S)}{U'(W^S)}\right) \\ +\left[\frac{p'(e_i)\frac{\partial e}{\partial c}U'(W^F)}{p(e_i)U'(W^S)}\left(1+\frac{(1-p(e_i))}{p(e_i)}\right)\right]\}\end{array} =$$

$$= \{-s\left[-A(W^F) + sA(W^S)\right] + \frac{p'(e_i)\frac{\partial e}{\partial c}U'(W^F)}{p(e_i)U'(W^S)}\left(\frac{1}{p(e_i)}\right)\} =$$

$$= \{-s\left[sA(W^S) - A(W^F)\right] + \left[\frac{\partial e}{\partial c}\frac{p'(e_i)}{(p(e_i))^2}\frac{U'(W^F)}{U'(W^S)}\right]\} \lesseqgtr 0$$

The first expression in curly brackets gives the negative risk aversion effect which makes more concave the indifference curve while the second positive term is the effort disincentive effect which renders more convex the indifference curve. The former is larger, the higher the degree of risk aversion, the more sensitive is the probability of success to the amount of collateral provided. However to simplify in figures 1 and 2, we design the indifference curves with a negative second derivatives because convexity due the dominance of the moral hazard impacts does not have significant implications for the existence and nature of equilibrium.

D) Proof of eq. (7):

We can again rewrite the slope of the indifference curve as:

$$\frac{dX}{dc} = M(w)P(w)$$

where $M(w) = -\frac{(1-p(e_i))}{p(e_i)}$ while $P(w) = \frac{U'(W^F)}{U'(W^S)}$. The curvature of the indifference curve with respect to change in wealth is then:

$$\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) = M(w)R'(w) + M'(w)R(w)$$

where $M(w)R'(w)$ captures the effect of risk preference effect while $M'(w)R(w)$ explains the moral hazard effect. First, let us solve $M(w)R'(w)$:

$$\begin{aligned}
M(w)R'(w) &= -\frac{(1-p(e_i))}{p(e_i)}\left[\frac{U''(W^F)U'(W^S) - U''(W^S)U'(W^F)}{(U'(W^S))^2}\right] = \\
&= -\frac{(1-p(e_i))}{p(e_i)}\left[\frac{U''(W^F)}{U'(W^S)} - \frac{U''(W^S)U'(W^F)}{(U'(W^S))^2}\right] = \\
&= -\frac{(1-p(e_i))}{p(e_i)}\frac{1}{U'(W^S)}\left[U''(W^F) - \frac{U''(W^S)U'(W^F)}{U'(W^S)}\right] = \\
&= -\frac{(1-p(e_i))}{p(e_i)}\frac{U'(W^F)}{U'(W^S)}\left[\frac{U''(W^F)}{U'(W^F)} - \frac{U''(W^S)}{U'(W^S)}\right]
\end{aligned}$$

Let us define $A(W)$ as the coefficient of decreasing absolute risk aversion, we can then rewrite $M(w)R'(w)$ as:

$$\begin{aligned}
M(w)R'(w) &= -\frac{(1-p(e_i))}{p(e_i)}\frac{U'(W^F)}{U'(W^S)}\left(A(W^S) - A(W^F)\right) \\
&= \frac{dX}{dC}\left(A(W^S) - A(W^F)\right) > 0
\end{aligned}$$

Since $W_1 > W_2$ and considering decreasing absolute risk aversion i.e. risk aversion decreases with wealth, $A(W^F) > A\left(W^S\right)$ and considering that by construction $\frac{dX}{dC}$ is negative, we can surely say that the effect of risk preferences $M(w)R'(w)$ is positive.

Then we can solve $M'(w)R(w)$:

$$\begin{aligned}
M'(w)R(w) &= \left[-\frac{-p'(e_i)\frac{\partial e}{\partial w}p(e_i)-(1-p(e_i))p'(e_i)\frac{\partial e}{\partial w}}{(p(e_i))^2}\right]\frac{U'(W^F)}{U'(W^S)} = \\
&= \left[\frac{p'(e_i)\frac{\partial e}{\partial w}}{(p(e_i))}+\frac{(1-p(e_i))p'(e_i)\frac{\partial e}{\partial w}}{(p(e_i))^2}\right]\frac{U'(W^F)}{U'(W^S)} = \\
&= \frac{p'(e_i)}{p(e_i)}\frac{\partial e}{\partial w}\left[1+\frac{(1-p(e_i))}{p(e_i)}\right]\frac{U'(W^F)}{U'(W^S)} = \\
&= \frac{p'(e_i)}{p(e_i)}\frac{\partial e}{\partial w}\left[\frac{U'(W^F)}{U'(W^S)}-\frac{dX}{dC}\right] = \\
&= -\frac{p'(e_i)}{p(e_i)}\frac{\partial e}{\partial w}\left[\frac{dX}{dC}-\frac{U'(W^F)}{U'(W^S)}\right] < 0
\end{aligned}$$

Therefore,

$$\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) = \frac{dX}{dC}\left(A(W^S)-A(W^F)\right) - \frac{p'(e_i)}{p(e_i)}\frac{\partial e}{\partial w}\left(\frac{dX}{dC}-\frac{U'(W^F)}{U'(W^S)}\right) \lesseqgtr 0$$

As shown, the sign of eq. (7) is uncertain due to the combination of the positive effect of risk aversion $\left(\frac{dX}{dC}\left(A(W^S)-A(W^F)\right)\right)$ and the negative moral hazard impact $-\frac{p'(e_i)}{p(e_i)}\frac{\partial e}{\partial w}\left(\frac{dX}{dC}-\frac{U'(W^F)}{U'(W^S)}\right)$.

D) Proof of eq. (8):
After some algebraic manipulations,

$$
\begin{aligned}
\frac{\partial}{\partial w}\left(\frac{dX}{dc}\right) &= s\left(A(W^S)-A(W^F)\right)-\frac{\partial e}{\partial w}\frac{p'(e_i)}{p(e_i)}\left(s-\frac{U'(W^F)}{U'(W^S)}\right)= \\
&= s(1-p(e_i))\left(A(W^S)-A(W^F)\right)-\frac{\partial e}{\partial w}p'(e_i)\frac{(1-p(e_i))}{p(e_i)}\left(s-\frac{U'(W^F)}{U'(W^S)}\right)= \\
&= s(1-p(e_i))\left(A(W^S)-A(W^F)\right)-\frac{\partial e}{\partial w}p'(e_i)\left(\left(\frac{(1-p(e_i))}{p(e_i)}\right)s+s\right)= \\
&= s(1-p(e_i))\left(A(W^S)-A(W^F)\right)-\frac{\partial e}{\partial w}p'(e_i)\frac{s}{p(e_i)} \\
&= s\left((1-p(e_i))\left(A(W^S)-A(W^F)\right)-\frac{\partial e}{\partial w}\frac{1}{p(e_i)\left(U(W^S)-U(W^F)\right)}\right)= \\
&= \frac{s}{p(e_i)\left(U(W^S)-U(W^F)\right)}\left(p(e_i)(1-p(e_i))\left(A(W^S)-A(W^F)\right)-\frac{\partial e}{\partial w}\right)
\end{aligned}
$$

The impact of moral hazard prevails if and only if:

$$
\frac{\partial e}{\partial w} > p(e_i)(1-p(e_i))\left(A(W^S)-A(W^F)\right)
$$

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