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Ordering policy for stock-dependent demand rate under progressive payment scheme: A comment

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Abstract: In a recent paper, Soni and Shah [2009. Ordering policy for stock-dependent demand rate under progressive payment scheme. International Journal of Systems Science 40, 81-89] developed a model for finding the optimal ordering policy for a retailer facing stock-dependent demand and a supplier offering a progressive payment scheme. In this note, we correct several errors in the formulation of the models of Soni and Shah and modify some assumptions to increase the model's applicability. Numerical examples illustrate the benefits of our modifications.

Introduction

In a recent paper, Soni and Shah (2009) developed a model for finding the optimal ordering policy for a retailer facing stock-dependent demand and a supplier offering a progressive payment scheme.[1] They assumed that in case the retailer pays before time M, the supplier does not charge any interest to the retailer, whereas in case the retailer pays between times M and N with $M < N$, the supplier charges an interest rate Ic_1. In case the retailer pays after time N, the supplier charges an interest rate Ic_2, with $Ic_2 > Ic_1$. In practice, the retailer often uses inventory as collateral to get a low-interest loan from a supplier (or a bank). However, in this case, the supplier is willing to provide a loan without any collateral or monthly payment. Revenues the retailer receives from selling products to the end customer may be deposited in an interest-bearing account until the account is completely settled[2], which leads to interest earnings at the rate of Ie. The authors assumed that in case the retailer is not able to settle the unpaid balance at time M (or N), s/he will settle as much of the unpaid balance as possible at these points in time. Afterwards, s/he continuously reduces the remaining debt by transferring incoming revenues to the supplier to minimize interest payments. Teng et al. (2011) recently extended Soni and Shah's model by including some additional aspects such as deterioration, limited capacity and non-zero ending inventory under profit maximization.

While assuming a progressive interest scheme offered to the retailer, Soni and Shah do not consider the case where $Ie > Ic_1$ in their model, although this case is not explicitly excluded in

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[1] Note that in contrast to what Soni and Shah state on pages 81 and 82 of their paper, demand in their model is exclusively stock-dependent and does not have a constant fraction.

[2] Thus, we do not consider investment decisions which are not related to the lot sizing problem.

the model assumptions. In the case where the interest rate of the retailer exceeds the interest rate charged by the supplier during the initial credit period (which may be the case in certain industries with a small number of powerful customers, see for example Ng et al., 1999 or Klapper et al., 2012), it is not rational from the retailer's perspective to settle the unpaid balance at time M. Instead, it would be better to keep the sales revenue in an interest-bearing account (see Summers and Wilson, 2002) and to settle the unpaid balance when the interest charged by the supplier exceeds the incomes from interest. We therefore add an assumption to the model and explicitly assume that the case $Ie > Ic_1$ may occur in addition to the other cases studied by Soni and Shah. However, we exclude the case $Ie > Ic_2$ to avoid scenarios where it would be rational for the retailer never to pay the supplier.

Depending on the ratio of the interest rates Ic_1 and Ie and the time when the retailer sells off the entire production lot, ten different cases may arise which are summarized in Figure 1. Note that U_1, U_2 and U_3 denote the unpaid balances at times M and N, respectively, and z the additional time which is required after times M or N to settle the unpaid balance completely. All cases will be discussed briefly in the following with reference to their treatment in the Soni and Shah (2009) paper. In addition, we will correct some errors contained in the original article. If not stated otherwise, we adopt the assumptions and notations used in Soni and Shah (2009) in the following.

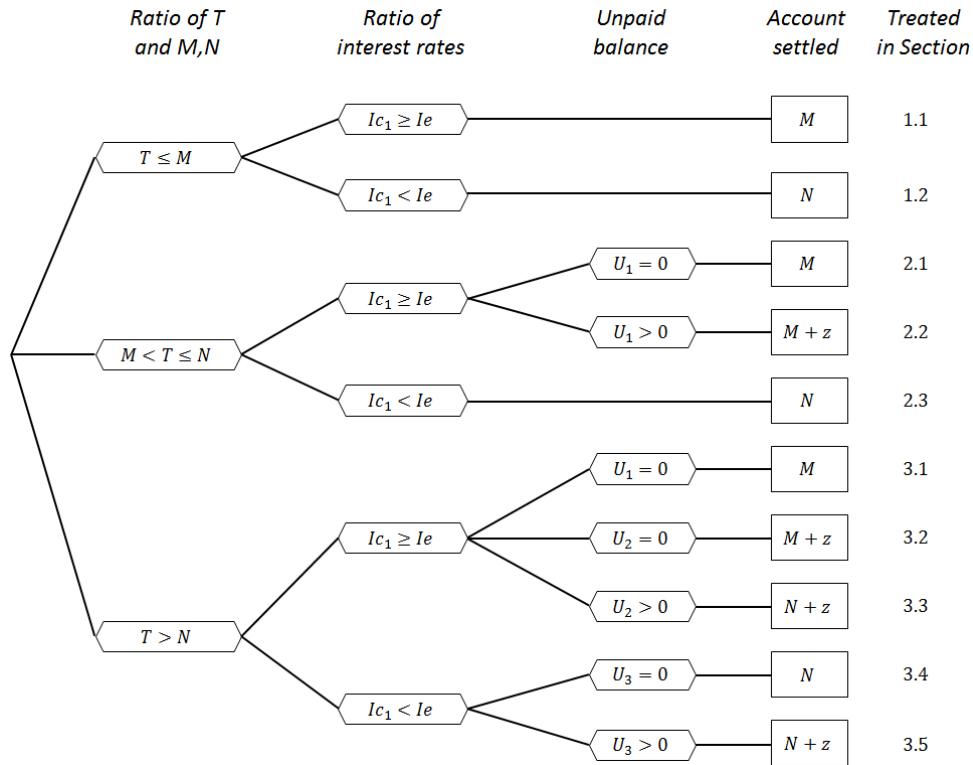| Ratio of T and M,N	Ratio of interest rates	Unpaid balance	Account settled	Treated in Section
$T \leq M$	$Ic_1 \geq Ie$		M	1.1
	$Ic_1 < Ie$		N	1.2
$M < T \leq N$	$Ic_1 \geq Ie$	$U_1 = 0$	M	2.1
		$U_1 > 0$	$M + z$	2.2
	$Ic_1 < Ie$		N	2.3
$T > N$	$Ic_1 \geq Ie$	$U_1 = 0$	M	3.1
		$U_2 = 0$	$M + z$	3.2
		$U_2 > 0$	$N + z$	3.3
	$Ic_1 < Ie$	$U_3 = 0$	N	3.4
		$U_3 > 0$	$N + z$	3.5

Figure 1: Cases for settling the unpaid balance

Modified model

Subcase 1.1: In Case 1 (which is Subcase 1.1 in our comment), Soni and Shah considered a scenario where the entire lot is sold off before the supplier starts charging an interest. If T denotes the point in time when the lot has been completely sold off, we have $T \leq M$. In this

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subcase, the retailer deposits the sales revenue in an interest-bearing account and settles the balance at time M. Correcting an error in the right-hand side of Soni and Shah's Eq. (7), the interest earned per year can be written as:

$$IE_{1,1} = \frac{PIe}{T}\left(\int_0^T R(t)t\,dt + Q(M-T)\right) = PIe\alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}T^{\beta/(1-\beta)}\left(M - \frac{T}{2-\beta}\right) \quad (1)$$

As the retailer settles the balance at time M, and therefore does not have to pay interest to the supplier (i.e., $IC_{1,1} = 0$), the total costs amount to:

$$TC_{1,1} = \frac{A}{T} - IeP\left(M - \frac{T}{2-\beta}\right)k_1 + \frac{hk_1}{2-\beta} \quad (2)$$

where $k_1 = \alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}T^{\beta/(1-\beta)}$.

The optimal solution to Eq. (2) is the solution of the following non-linear equation (provided that the second derivation of Eq. (2) with respect to T is greater than zero for all T):

$$\frac{dTC_{1,1}}{dT} = -\frac{A}{T^2} + \frac{IePk_1}{2-\beta} - \frac{IeP\left(M - \frac{T}{2-\beta}\right)\beta k_1}{T(1-\beta)} + \frac{h\beta k_1}{T(1-\beta)(2-\beta)} \quad (3)$$

Subcase 1.2: For $T \le M$ and $Ie > Ic_1$, the retailer achieves a financial benefit from postponing the refund and keeping the sales revenue in an interest-bearing account until time N. Between times M and N, s/he has to pay interest to the supplier. However, due to $Ie > Ic_1$, the interest earned exceeds the interest paid within this period. Similar to Subcase 1.1, the interest earned per year can be calculated as:

$$IE_{1,2} = \frac{PIe}{T}\left(\int_0^T R(t)t\,dt + Q(N-T)\right) = PIe\alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}T^{\beta/(1-\beta)}\left(N - \frac{T}{2-\beta}\right) \quad (4)$$

The overall interest cost between M and N, on the other hand, amount to:

$$IC_{1,2} = \frac{Ic_1}{T}CQ(N-M) \quad (5)$$

The total costs are thus calculated as:

$$TC_{1,2} = \frac{A}{T} + CIc_1(N-M)k_1 - IeP\left(N - \frac{T}{2-\beta}\right)k_1 + \frac{hk_1}{2-\beta} \quad (6)$$

where $k_1 = \alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}T^{\beta/(1-\beta)}$.

The optimal solution to Eq. (6) is the solution of the following non-linear equation (provided that the second derivation of Eq. (6) with respect to T is greater than zero for all T):

$$\frac{dTC_{1,2}}{dT} = -\frac{A}{T^2} - \frac{CIc_1(N-M)\beta k_1}{T(\beta-1)} + \frac{IePk_1}{2-\beta} - \frac{IeP\left(N - \frac{T}{2-\beta}\right)\beta k_1}{T(1-\beta)} + \frac{h\beta k_1}{T(1-\beta)(2-\beta)} \quad (7)$$

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Subcase 2.1: In the case where $M < T \leq N$ and $Ie \leq Ic_1$, the retailer settles as much of the unpaid balance as possible at time M to minimize interest payments. In the first subcase, it is assumed that the sum of sales revenue and interest earned at time M is sufficient to settle the unpaid balance, i.e. $U_1 = 0$, where U_1 is the buyer's debt at time M. The interest earned until time M is formulated as follows (note that this formulation corrects an error in the right-hand side of Soni and Shah's Eq. (11)):

$$IE_{2,1} = \frac{PIe}{T}\int_0^M R(t)t\,dt = \frac{PIe}{T(2-\beta)}\alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}\left(T^{(2-\beta)/(1-\beta)}(1-\beta) - (T-M)^{1/(1-\beta)}(M+T(1-\beta))\right)$$

$$(8)$$

As the retailer does not have to pay interest to the supplier in this subcase (i.e. $IC_{2,1} = 0$), the total costs amount to:

$$TC_{2,1} = \frac{A}{T} + \frac{hk_2}{T(2-\beta)} + \frac{IeP(1-\beta)k_2}{2-\beta} - \frac{IeP(T-M)^{1/(1-\beta)}(M+T(1-\beta))k_2}{(2-\beta)T^{(2-\beta)/(1-\beta)}} \tag{9}$$

where $k_2 = \alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}T^{1/(1-\beta)}$.

The optimal solution to Eq. (9) is the solution of the following non-linear equation (provided that the second derivation of Eq. (9) with respect to T is greater than zero for all T):

$$\frac{dTC_{2,1}}{dT} = -\frac{A}{T^2} + \frac{h\beta k_2}{T^2(1-\beta)(2-\beta)} + \frac{IeP(1-\beta)k_2}{2-\beta} - \frac{IeP(T-M)^{1/(1-\beta)}(M+T(1-\beta))k_2}{(2-\beta)T^{(2-\beta)/(1-\beta)}} + \frac{IeP(1-\beta)(T-M)^{1/(1-\beta)}k_2}{(2-\beta)T^{(2-\beta)/(1-\beta)}} - \frac{2IeP(1-\beta)^{\beta+1}k_2}{(2-\beta)T} - \frac{IeP\beta k_2}{2-\beta} + \frac{IeP(T-M)^{\beta/(1-\beta)}(M+T(1-\beta))k_2}{(2-\beta)T^{(2-\beta)/(1-\beta)}} + \frac{IeP(T-M)^{1/(1-\beta)}\beta(M+T(1-\beta))k_2}{(2-\beta)(1-\beta)T^{(2-\beta)/(1-\beta)}}$$

$$(10)$$

Subcase 2.2: In contrast to Subcase 2.1, we now consider the case where the sum of sales revenue and interest earned at time M is not sufficient to settle the balance completely, i.e. $U_1 > 0$. Thus, the retailer has to pay interest to the supplier. Interest earned is the same as the one given in Eq. (8). In calculating the unpaid balance U_1, Soni and Shah assumed that $U_1 = CQ - (PR(M)M + IE_2)$. $R(t)$, in this context, denotes the stock-dependent demand rate at time t. Since the demand rate decreases in t due to a decreasing inventory level, we note that $PR(M)M$ underestimates the sales revenue of the retailer, since $R(M) < R(M-\Delta)$ for $\Delta > 0$. As a consequence, U_1 has to be reformulated as follows:

$$U_1 = CQ - \left(P\int_0^M R(t)\,dt + PIe\int_0^M R(t)t\,dt\right) \tag{11}$$

Furthermore, the authors mention that the "retailer will have to pay interest on un-paid balance […] at the rate of Ic_1 at time M to the supplier" (cf. p. 84). However, we note that after the account has been partially settled at time M, the retailer has no money left to pay interests in advance. We therefore modify Soni and Shah's hypothesis and assume that in case $U_1 > 0$ and $Ie \leq Ic_1$, the retailer transfers each dollar s/he earns after time M directly to the supplier to minimize interest payments (see Goyal et al., 2007 for a similar assumption). For the case where the

unpaid balance cannot be settled at time M, but before time N, it follows that interest paid as given in Eq. (17) of the Soni and Shah-paper can be reformulated as follows:

$$IC_{2,2} = \frac{Ic_1}{T} \int_M^{M+z} U_1 - PR(t)(t-M)\mathrm{d}t \tag{12}$$

where $M+z$ denotes the point in time when the unpaid balance has been completely settled, with $z > 0$ and $M+z < T$. The total costs for this case amount to:

$$TC_{2,2} = \frac{A}{T} + \frac{hk_2}{T(2-\beta)} + \frac{Ic_1}{T} \int_M^{M+z} U_1 - PR(t)(t-M)\mathrm{d}t + \frac{IeP(1-\beta)k_2}{2-\beta} - \frac{IeP(T-M)^{1/(1-\beta)}(M+T(1-\beta))k_2}{(2-\beta)T^{(2-\beta)/(1-\beta)}} \tag{13}$$

where $k_2 = \alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}T^{1/(1-\beta)}$. Due to the indefinite integral, we are unable to calculate an optimality condition for Subcase 2.2 explicitly. However, we note that the value of z can be approximated numerically with arbitrary precision (e.g. with the help of the bisection method). This permits us to calculate a near-optimal solution numerically for this subcase.

Subcase 2.3: This subcase (i.e. the case where $M < T \leq N$ and $Ie > Ic_1$) is identical to Subcase 1.2.

Subcase 3.1: This subcase (i.e. the case where $T > N$, $Ie \leq Ic_1$ and $U_1 = 0$) is identical to Subcase 2.1.

Subcase 3.2: This subcase (i.e. the case where $T > N$, $Ie \leq Ic_1$, $U_1 > 0$ and $U_2 = 0$) is identical to Subcase 2.2.

Subcase 3.3: In this subcase, with $T > N$ and $Ie \leq Ic_1$, the retailer is not able to pay off the total purchase cost at M or N. Thus, s/he will settle as much of the balance as is possible at times M and N. Between times M and N, the sales revenue is kept in an interest-bearing account, and the supplier charges interest on the outstanding balance U_1 with interest rate Ic_1. Afterwards, as in Subcase 2.2, the retailer transfers each dollar s/he earns directly to the supplier who charges interest on the gradually reducing unpaid balance U_2 at the interest rate Ic_2. As the retailer partially settles the account in M and N, s/he is able to realize interest earnings in the period $[0, N]$, which can be calculated as:

$$IE_{3,3} = \frac{PIe}{T}\left(\int_0^M R(t)t\mathrm{d}t + \int_M^N R(t)(t-M)\mathrm{d}t\right) \tag{14}$$

The unsettled balance U_2 (at time N) calculated by Soni and Shah again underestimates the sales revenue of the retailer. Further, while estimating the interest earnings between times M and N, the authors neglected the time the revenue is kept in the account. Therefore, U_2 has to be reformulated as follows:

$$U_2 = U_1\big(1 + Ic_1(N-M)\big) - \left(P \int_M^N R(t)\,\mathrm{d}t + PIe \int_M^N R(t)(t-M)\,dt\right) \tag{15}$$

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where U_1 is the unpaid balance at time M as given in Eq. (11). Consequently, the interest payable per year, $IC_{3,3}$, is given as:

$$IC_{3,3} = \frac{Ic_1}{T}U_1(N-M) + \frac{Ic_2}{T}\int_N^{N+z}U_2 - PR(t)(t-N)\,\mathrm{d}t \tag{16}$$

where $N+z$ denotes the point in time when the unpaid balance has been settled, with $z > 0$ and $N+z \le T$.

The objective function for Subcase 3.3 has the same structure and solution procedure as the one given in Eq. (13), with the exceptions that $IC_{2,2}$ needs to be substituted by $IC_{3,3}$ and that the interests earnings $IE_{3,3}$ have to be considered. Again, a near-optimal solution can be calculated numerically for this subcase.

Subcase 3.4: If the interest rate of the retailer Ie exceeds the interest charges of the supplier for the first credit period, Ic_1, s/he will again not settle the account before N. Instead, the retailer keeps the sales revenues between times M and N in an interest-bearing account. As the unpaid balance U_3 is assumed to be 0 in this subcase, the account is completely settled at time N. Thus, the interest earned is given as:

$$IE_{3,4} = \frac{PIe}{T}\int_0^N R(t)t\,\mathrm{d}t = \frac{IePa^{1/(1-\beta)}(1-\beta)^{(2-\beta)/(1-\beta)}}{T(2-\beta)}\left(T^{(2-\beta)/(1-\beta)} - (T-N)^{1/(1-\beta)}\left(N + T(1-\beta)\right)\right) \tag{17}$$

The interest charges in the period $[M, N]$ amount to:

$$IC_{3,4} = \frac{Ic_1}{T}CQ(N-M) \tag{18}$$

Thus, the total costs for this subcase are formulated as:

$$TC_{3,4} = \frac{A}{T} + CIc_1(N-M)k_1 + \frac{hk_1}{2-\beta} - \frac{\beta-1}{\beta-2}\frac{IeP\alpha}{T}\left((N-T)(N+T-T\beta)k_4^\beta - T^2(\beta-1)k_3^\beta\right) \tag{19}$$

where $k_1 = \alpha^{1/(1-\beta)}(1-\beta)^{1/(1-\beta)}T^{\beta/(1-\beta)}$, $k_3 = (\alpha(1-\beta)T)^{\frac{1}{1-\beta}}$ and $k_4 = \left(\alpha(1-\beta)(T-N)\right)^{\frac{1}{1-\beta}}$.

The optimal solution to Eq. (18) is the solution of the following non-linear equation (provided that the second derivation of Eq. (18) with respect to T is greater than zero for all T):

$$\frac{dTC_{3,4}}{dT} = -\frac{A}{T^2} + CIc_1(N-M)T^{\frac{2\beta-1}{1-\beta}}(1-\beta)^{\frac{\beta}{1-\beta}}\beta + \frac{hT^{\frac{2\beta-1}{1-\beta}}\alpha^{\frac{1}{1-\beta}}(1-\beta)^{\frac{\beta}{1-\beta}}\beta}{2-\beta} -$$
$$\frac{IeP\alpha(\beta-1)((N-T)(N+T-T\beta)k_4^\beta-T^2(\beta-1)k_3^\beta)}{T^2(2-\beta)} + \frac{IePT^{\frac{1+\beta}{1-\beta}}\alpha(1-\beta)}{(2-\beta)k_3k_4}\left((N-T)^{\frac{\beta}{1-\beta}}(T^{\frac{\beta}{\beta-1}} - \right.$$

$$NT^{\frac{1}{\beta-1}})\alpha^{\frac{1}{1-\beta}}(T^{\frac{\beta}{\beta-1}}(\beta-1)-NT^{\frac{1}{\beta-1}})(1-\beta)^{\frac{\beta}{1-\beta}}\beta k_3 k_4^{\beta}+T^{\frac{1}{\beta-1}}(NT^{\frac{1}{\beta-1}}\beta-2T^{\frac{\beta}{\beta-1}}(\beta-$$

$$1))k_3 k_4^{1+\beta}+T^{\frac{\beta}{\beta-1}}(\beta-1)k_3^{\beta}k_4(\alpha^{\frac{1}{1-\beta}}(1-\beta)^{\frac{\beta}{1-\beta}}\beta+2T^{\frac{1}{\beta-1}}k_3)) \tag{20}$$

Subcase 3.5: For the case where $Ie > Ic_1$ and where the retailer is unable to settle the balance completely at time N, the account is partially settled at time N and hereafter the unpaid balance is continuously reduced by sales revenues until it is completely settled. The interest earnings until time N are the same as those given in Eq. (17).

In addition, the unpaid balance at time N equals:

$$U_3 = CQ\big(1+Ic_1(N-M)\big)-\Big(P\int_0^N R(t)\,dt + IeP\int_0^N R(t)t\,dt\Big) \tag{21}$$

The interest charges amount to:

$$IC_{3,5}=\frac{Ic_1}{T}CQ(N-M)+\frac{Ic_2}{T}\int_N^{N+z}U_3-PR(t)(t-N)\,dt \tag{22}$$

where $N+z$ denotes the point when the unpaid balance has been settled, with $z > 0$ and $N+z \le T$. The objective function for Subcase 3.5 has the same structure and solution procedure as the one given in Eq. (13), with the exception that $IC_{2,2}$ needs to be substituted by $IC_{3,5}$ and the interest earning $IE_{3,4}$ have to be considered. Again, a near-optimal solution can be calculated numerically for this subcase.

Numerical examples
To illustrate the behavior of our model, we consider the parametric values shown in Table 1 and the payment policies of the retailer introduced above. The numerical examples (cf. Table 2) indicate that:

1. For a fixed consumption rate, an increase in the first credit period has only minor influences on the order quantity and the length of the order cycles. The total costs, in turn, are reduced as M adopts higher values. An increase in the second credit period results in higher order quantities, a longer order cycle length and lower total costs.
2. An inverse interest structure with $Ie > Ic_1$ does not affect the lot size policy itself. However, it affects the optimal payment policy of the retailer, who may choose a different point in time to settle the balance. In contrast to the model of Soni and Shah (cf. TC_1 in Table 2), the presented payment policy (cf. TC_2 in Table 2) may reduce the total costs of the buyer.

Table 1: Model parameters

α	=	100	first parameter of the demand function
β	=	0.30	second parameter of the demand function
A	=	100	ordering cost per order
C	=	20	unit purchase cost

h =	0.20	inventory holding cost per unit and year	
Ic_1 =	0.10	interest rate per year for the first credit period	
Ic_2 =	0.18	interest rate per year for the second credit period	
Ie =	0.14	interest rate on deposits for the retailer	
M =	15	first permissible credit period	
N =	30	second permissible credit period	
P =	30	unit selling price	

Table 2: Effect of M and N on decision parameters

$M\rightarrow$ $N\downarrow$	15/365	20/365	25/367
30/365	$T = 0.4822$ $Q = 152.52$ $R = 451.85$ $TC_1 = 303.39$ $TC_2 = 301.74$	$T = 0.4877$ $Q = 155.00$ $R = 454.04$ $TC_1 = 294.14$ $TC_2 = 292.70$	$T = 0.4932$ $Q = 157.49$ $R = 456.22$ $TC_1 = 284.52$ $TC_2 = 283.63$
35/365	$T = 0.4932$ $Q = 157.49$ $R = 456.22$ $TC_1 = 297.28$ $TC_2 = 295.16$	$T = 0.4932$ $Q = 157.49$ $R = 456.22$ $TC_1 = 288.23$ $TC_2 = 286.15$	$T = 0.4986$ $Q = 160.00$ $R = 458.39$ $TC_1 = 278.78$ $TC_2 = 277.06$
40/365	$T = 0.4877$ $Q = 155.00$ $R = 454.04$ $TC_1 = 291.66$ $TC_2 = 289.09$	$T = 0.4932$ $Q = 157.49$ $R = 456.22$ $TC_1 = 282.69$ $TC_2 = 279.99$	$T = 0.4986$ $Q = 160.00$ $R = 458.39$ $TC_1 = 273.37$ $TC_2 = 270.88$

Conclusion

In this comment, we corrected some errors in a recent paper of Soni and Shah (2009) and modified some of its assumptions to increase the model's applicability. One important modification is that we assumed that the interest charged by the supplier in the first credit period, Ic_1, may be lower than the interest earned by the buyer in this period, Ie. Such a scenario may occur in practice, for example if buyer and supplier have access to different sources of funding or different investment opportunities, which may result in different interest rates that are used at both actors. In numerical examples, we illustrated the behavior of our model and showed that the optimal payment policy, which is dependent on the current interest structure, may lead to lower cost without affecting the lot-size policy itself.

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