Trading in Electronic Markets: The Challenges of Imperfect Liquidity and Reduced Pre-Trade Transparency

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Declaration

I hereby declare that no degree or other qualification has been granted for any work included in this thesis and that the thesis is my own original research work, except where specified in the thesis.

I agree to deposit this thesis in the University Institutional Repository and submit an electronic copy to be made available through EThOS.

To my mother who makes it all possible.

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Abstract

This thesis is motivated by the progressive expansion of electronic markets with reduced pre-trade transparency and the collateral liquidity effects. In this thesis I develop three independent theoretical models and explore the repercussions of weak market liquidity and transparency.

First, I approach the issue of limited liquidity through the optimal order placement problem of a risk-averse trader in a continuous time context and introduce a random delay parameter, which defers limit order execution and characterises market liquidity. This framework demonstrates that imperfect liquidity explains order clustering in the proximity of best quotes and the existence of the bid-ask spread. The distribution of expected time-to-fill of limit orders conforms to the empirically observed distribution of trading times, and its variance decreases with liquidity. Finally, two additional stylised facts are rationalised in this model: the equilibrium bid-ask spread decreases with liquidity, but increases with agents' risk aversion.

My second framework adjoins the few theoretical attempts in the literature to challenge investors' incentives to participate in opaque trading environments. Through a real option approach I justify how market opacity can encourage liquidity provision, which, in turn, supports the empirical evidence on the proliferation of such trading venues. I demonstrate that transparency in conjunction with liquidity determine traders' eagerness to supply liquidity to the opaque market, and that once the trader enters the opaque market, he commits to trade relatively quickly. Furthermore, error analysis reveals that impatient traders are highly likely to pass over favourable trade execution offered in the opaque market precisely because of imperfect clarity of information signals, while a prior optimistic bias prompts the trader to submit his limit order sooner.

Lastly, as a complement to static inference on individual level, I establish an artificial market with heterogeneous agents and distinct transparency regimes that replicates the long memory properties and empirical order flow patterns. The results suggest that full quote transparency incurs substantial transaction costs and dampens trading activity, while exogenous restriction of displayed depth up to several quotes does not alter significantly market performance. The core implication of this model is that the endogenous restriction of displayed quote depth by means of iceberg orders improves market quality in multiple dimensions.

This thesis contributes to the microstructure domain by providing a theoretical support to the benefits of market opacity as well as its downsides. The research outcomes of this thesis are, therefore, relevant from a regulatory standpoint.

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Chapter 1

Introduction

I sometimes think that speculation must be an unnatural sort of business, because I find that the average speculator has arrayed against his own nature. The weaknesses that all men are prone to are fatal to success in speculation usually those very weaknesses that make him likable to his fellows or that he himself particularly guards against in those other ventures of his where they are not nearly so dangerous as when he is trading in commodities or stocks.

– Edwin Lefèvre, *Reminiscences of a Stock Operator.*

If you have too much respect for facts, especially when they refute your conjectures, you will go on with pre-naive trial-and-error and look for another conjecture. But if you have a better heuristic, you at least try to ignore the adverse observational test, and try a test by thought-experiment [...].

– Imre Lakatos, *Proofs and Refutations.*

In this thesis I develop three comprehensive frameworks that address repercussions of trading rules imposed in financial markets and the associated strategic behaviour of market participants. The issues of my primary interest are pre-trade quote transparency and imperfect liquidity in electronic markets. This chapter renders a brief overview of these problems put in context, explains the main motivation behind my research and locates its niche in financial economics. Following this I summarise my contributions to the existing literature on this subject and close the chapter with a path walk of the remaining parts of the thesis.

1.1 Motivation

This thesis shares the principal objective of all market microstructure studies in that it explores the routes by which the architecture of financial markets perturbs price discovery, trading activity, and market quality in a broad sense. In my study I justify the relevance of market liquidity and transparency regimes, which arguably emerge as yet more intricate in the milieu of electronic trading systems, to market performance. My narrative commences with a revision of key market microstructure aspects and gradually leads to a discussion about the roles that weak liquidity and market transparency rules play in asset price formation, in the promotion of competition among traders, and in creating incentives to provide sustainable market depth.

1.1.1 Background

In contrast with classical economics which analyses the equilibrium prices at which demand equals supply, market microstructure exploits how specific trading mechanisms guide the evolution of prices over time. While a unique fundamental price remains a useful theoretical concept, much understanding of short run market fluctuations and adjustments is gained by scrutinising the minutiaes of the asset exchange process implied by a concrete market structure. Several expansive surveys, that overview findings and methodologies in market microstructure research, have been accomplished to date. The pioneering work of O'Hara (1997) consolidated inventory models, followed by extensive overviews of more advanced models by Madhavan (2000) and Biais et al. (2005), and Parlour and Seppi (2008) with a specific emphasis on limit order markets. In addition, Hasbrouck (2006) revised empirical microstructure literature, and Harris (2003) collected microstructure insights for practitioners and regulators. These surveys and empirical knowledge emanate five major properties that mould the microstructure of every financial market:

- trade intermediation,
- time and frequency of trading,
- location,
- transparency, and
- rules and regulations.

Trade Intermediation

There are three principal methods used on trading venues to arrange the exchange of assets between agents: a quote-driven, a brokered, and an order-driven system. In quote-driven markets there is a designated specialist, one or several, who aggregates market demand and supply and intermediates all transactions between traders. Brokered trading assumes that brokers assist the matching of counterparties without participating in their transactions. In non-intermediated order-driven markets buyers and sellers find each other by adopting certain order precedence rules that ultimately determine the prices at which trades are executed.

While in a quote-driven system a market maker is responsible for liquidity provision and has to maintain his inventory position for this reason, in order-driven and brokerage systems all market participants jointly generate liquidity necessary for smooth functioning of the market. The majority of modern financial markets are either hybrids, with the prominent examples of NYSE and NASDAQ, or non-intermediated electronic order matching systems. Top world exchanges rely extensively on the electronic order book. According to the data collected by the World Federation of Exchanges, the cumulative value of shares transacted on electronic platforms has not declined dramatically ever since its rapid surge and a recent global boom in 2003-2009. Typically, the execution of trades on each stock exchange is achieved in either of the three modes: (i) transactions are effected in the electronic order book as a result of automatic order match according to precise exchange priority rules, (ii) negotiated deals are executed on an alternative platform with the price confirmed by both the buyer and the seller, and (iii) over-the-counter trades are executed off the exchange and reported a posteriori. Figure 1.1 depicts the split of the shares turnover between three modes of trade execution on the basis of year-to-date estimates in December 2012, with the exchanges ranked by market capitalisation. This chart highlights that trades matched automatically through electronic limit order book exhibit remarkable prevalence across all major exchanges, annihilating the role of market makers.

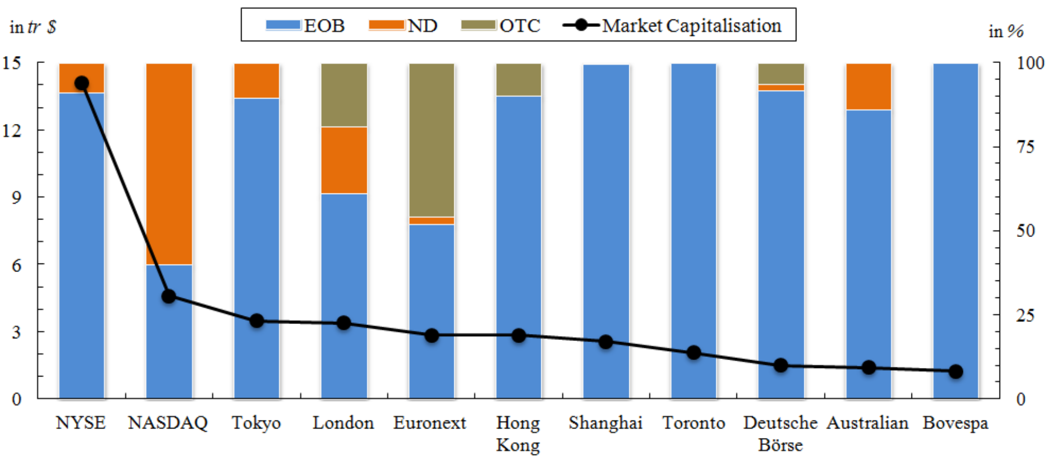


Figure 1.1: The share turnover split between trades matched automatically in the electronic order book (EOB), negotiated (ND) and over-the-counter (OTC) deals for the largest global exchanges by market capitalisation in December 2012. Source: World Federation of Exchanges.

The distinction between quote-driven markets with inventory positions and competitive behaviour, on the one hand, and order-driven markets with strategic behaviour, on the other, motivated the change in both perception and methodologies used in market microstructure research. As Biais et al. (2005, p.220) note, the focus of analysis has shifted from prices to order flow and limit order book shape information. My study targets primarily the latter type of market organisation. Given the heterogeneity and multitude of traders and their trading styles, market dynamics induced by order-driven trade execution mechanism are highly complex. Moreover, the diversity of operational market phases imposed by exchanges tends to exacerbate this complexity.

Time and Frequency of Trading

A typical schedule of a trading day can be traced from the example of the New York Stock Exchange Arca (NYSE Arca) platform for trading in equities (Table 1.1). Each day on NYSE Arca Equities begins at 3:30 a.m. with the 29-minute session when limit orders are submitted to the market and saved in the book. During this period traders can cancel or modify their orders up until the last minute, or keep these orders in the book till 4:00 a.m otherwise. Trading is initiated by the Opening Session with a one-minute Auction Freeze, when all accumulated limit orders are matched, and a 5.5-hours Opening Auction, which concludes with a one-minute Market Order Freeze whereabout market orders or auction limit orders can no longer be amended and new orders are not accepted to avoid price distortions. At 9:30 a.m. the main trading session opens and active real-time trading is carried out until 3:59 p.m. Closing Auction Freeze from 3:59 p.m. to 4:00 p.m. bans cancellations of day orders and forbids order placement on the side of current imbalance to prevent last minute price manipulations. Next, with the Closing Auction Run the market end-of-the-day information about prices and trades is published. Finally, all limit orders that arrived after closing or expired by the end of the day are cancelled at 8:00 p.m. and cross orders are matched through a portfolio crossing system.

The total operation time on the NYSE Arca Equities is distributed rather evenly between the principal active trading session that occupies 6.5 hours, 6 hours for price discovering sessions and preparation for the next opening, and the remaining 4 hours dedicated to market clearing process. Most of the venues are closed for trading overnight; some markets are open for longer with a maximum of 11 hours across Europe, others – for fewer hours per day. Many Asian markets still close for a lunch break, for example: Tokyo Stock Exchange and Hong Kong Stock Exchange have two core trading sessions per day separated by a one-hour interval. However, daily schedules of most markets with several market phases are fairly similar. For example, Figure 1.2 indicates that although the distributions of time varies between the New York Stock Exchange and the London Stock Exchange, the active trading period is only 2 hours longer in the latter.

Daily trading schedules are designed to accommodate efficiently overnight arrivals of information. Also, the coordination of market opening times around the globe allows to trade 24 hours a day without interruption and incorporate latest market, macroeconomic and business news timely.

Table 1.1: Market phases at the NYSE Arca Equities. Source: NYSE Euronext.

Pre-Opening Session		3:30–3:59 a.m.
3:30–3:59 a.m.	Limit Orders can be entered and queued	
Opening Session		4:00–9:30 a.m.
4:00 a.m.	Opening Auction Freeze Period	
4:00–9:29 a.m.	Opening Auction	
9:29–9:30 a.m.	Market Order Freeze Period	
9:30 a.m.	Market Order Auction	
Core Trading Session		9:30–4:00 p.m.
9:30–3:59 p.m.	Active Trading Session	
3:59–4:00 p.m.	Closing Auction Freeze Period	
4:00 p.m.	Closing Auction Run	
Extended Hours		4:00–8:00 p.m.
8:00 p.m.	Limit Orders entered after 4:00 p.m. are cancelled	
8:01 p.m.	Portfolio Crossing System (PCS)	

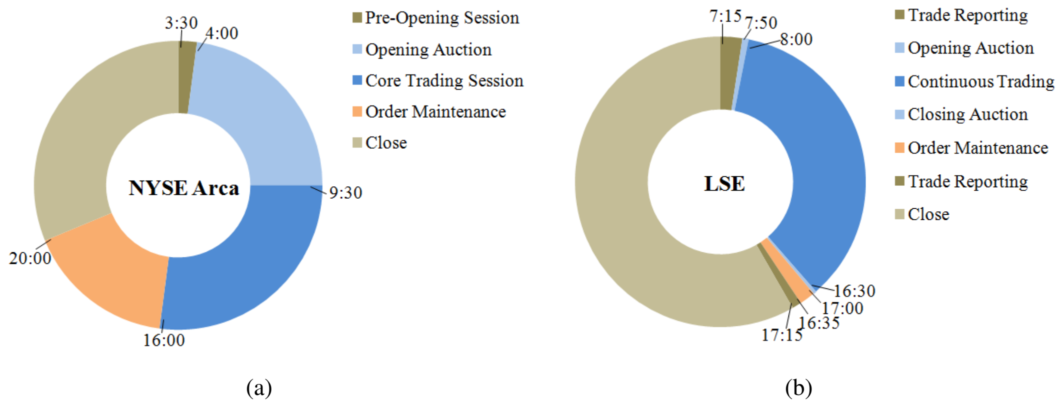


Figure 1.2: Market phases on a regular trading day (a) on the New York Stock Exchange Arca (NYSE Arca), and (b) on the London Stock Exchange (LSE).

Daily schedules of markets are important since, for instance, they generate intraday U-shaped pattern and day-of-the-week seasonality in the intensity of trading activity and trades aggressiveness. For convenience of theoretical analysis I deliberately focus, in most theoretical microstructure models, on the core asset exchange period – a continuous trading session. This abstraction implies that the set of order placement and execution rules is fixed throughout trading.

Location

The third key microstructure determinant is market location. Trading in financial markets traditionally took place on physical trading floors of local exchanges with people shouting out their orders. Nowadays, new communication technologies have given rise to virtual trading venues with rapid information transmission which enables traders to receive up-to-date market feedback and act upon it instantaneously. Trading in electronic markets offers substantially higher speed of order processing and execution and permits stronger participation. Assured by obvious benefits of computerisation, such as availability, convenience and minimal efforts required from all trading parties, lower transaction and maintenance costs and enhanced market efficiency, many exchanges incline towards online electronic matching systems to boost turnovers.

Notwithstanding the exact operation mode of a market, its location does not need to be unique. If a particular security is traded exclusively on one exchange, the market for this security is consolidated. However, each given asset can be potentially listed on multiple organised exchanges, be they physical or virtual, or be transferred in over-the-counter deals, thereby causing market fragmentation. Computing and communication innovations accelerate the ease of creating new markets and in this manner intensify competition between various trading service providers, which, in turn, inevitably accentuates market fragmentation issues.

Summarising the topic of fragmentation, Harris (2003) reasons that the desirability of fragmentation is ambiguous. A consolidated market eliminates competition between various trading venues to provide services and trading rules that are attractive to all and at better costs. However, a single market certainly cannot satisfy the needs of every trader, whereas diversity of markets allows agents to select most suitable venues to realise their trading intentions. At the same time a fragmented market entails lack of competition among traders and demotivates specialists to secure best prices. The research report *Navigating Liquidity 6* (Lehalle et al., 2012) attributes the creation of the Markets in Financial Instruments Directive (MiFID) in November 2007 to the milestones in the regulation of fragmentation: it established a legal framework for competition between traditional and modern trading venues. On the practitioners side, there has been made a number of attempts to measure and diagnose market fragmentation. Among widely accepted quantitative indicators counts the Cheuvreux Fragmentation Efficiency Index, that uses a physics formula of entropy (i.e. disorder in a system) to evaluate the level of fragmentation that a given security attains; and Fragulator® (Fidessa Fragmentation Index) is a tool that offers a single number estimate of the degree of market fragmentation. The latter tool assists investors in measuring the dispersion of liquidity and allows to differentiate between various combinations of security split across primary and alternative venues. However, regardless of investors' ability to incorporate the extent of market fragmentation into their strategies and balance between multiple trading venues using smart order routing and computer algorithms, all traders are naturally bound to flock to the highest liquidity and produce an order flow externality. In other words, all investors, despite tremendous heterogeneity of trading motives, prefer to participate in the most liquid markets.

Transparency

In light of fragmentation, both regulators and businesses are interested in understanding which market conditions imply fair competition for liquidity and draw additional liquidity, and how liquidity is to be assessed. Opaque markets that pertain limited transparency regarding the state of the market and executed trades, play a special role in this discussion. A thorough study by Board et al. (2002) in the recent book *"Transparency and Fragmentation"* investigates this link.

Within the spectrum of opaque markets, some exchanges display only the depth at a few best quotes in the limit order book, whereas other exchanges endogenise the regulation of transparency by permitting traders to submit iceberg orders that conceal partly or entirely the limit order size. Iceberg orders are designed primarily for large institutions and exceptionally wealthy individuals that employ block trading and thereby require delicate cost and exposure management. Certain new opaque markets, for instance, display only partial information about incoming orders and current market depth striving to gain a competitive advantage over older existing exchanges that have by now attracted many versatile investors and to circumvent the positive order flow externality that those older exchanges exercise. Moreover, special attention has been drawn to dark markets where the limit order book is entirely hidden from trading agents and prices with corresponding volumes are "known" only to the electronic system. The anonymity of trades in dark pools aims to prevent explicit price manipulation. According to Fidessa datasource (Fidessa, 2012), the cumulative volume transacted on the European dark venues has increased more than tenfold in the past five years and continues a stable positive trend. The proliferation of dark and semi-transparent venues gives rise to a variety of interesting questions: how does the market benefit from reduced transparency rules, what kind of agents are enticed by opacity and what are their gains, how frequently traders are misled by excessive uncertainty in these markets?

Rules and Regulation

Transparency rules compose only one brick of the ultimate market microstructure pillar – trading rules and regulations. On the one hand, markets have plentiful freedom to choose the preferred operation mode and order handling rules: be it physical floor exchange or an on-screen platform, with an assigned dealer to mediate transactions or auction-based trade execution. To give an example, orders waiting in the limit orders book can be allocated according to a *price-time* priority protocol, whereby the earliest order at the matching price is fully executed before switching to the next order that was placed at the same price later. Alternatively, some exchanges favour a *pro rata* priority protocol: when a market order arrives each agent queuing at the best quote gets a fill proportionally

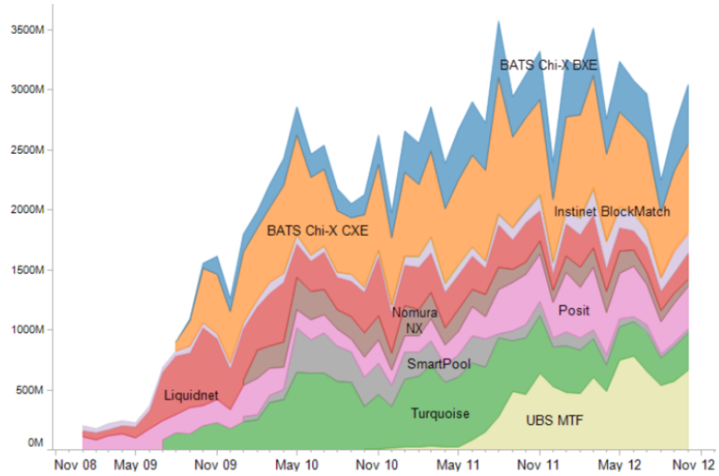


Figure 1.3: The estimates of the volumes traded in European dark pools. Source: Fidessa (2012).

to his total order size. Exchanges establish their rules based on specific trader profiles they intend to engage. All along empirical evidence traces a persistent link between the choice of trading principles and ensuing market quality in terms of liquidity, transaction costs, and other characteristics. Therefore, it is essential that competent regulatory authorities give prescriptions to trading venues and impose certain constraints on them in order to promote a fair and competitive trading environment and enhance market efficiency and economic welfare. The directives devised by regulators are, in turn, guided by substantial economic research.

In this thesis I contemplate two broad issues recognised in market microstructure research: liquidity and transparency. More precisely, my interest lies in departures from perfect liquidity and transparency, their origins and inflicted consequences. These two issues, as becomes apparent after a closer examination, are tightly intertwined. This thesis, therefore, sets out to examine liquidity and transparency from several angles, and the following subsections give an initial outline of the concepts and methodologies adopted in my work.

1.1.2 Imperfect Liquidity

Liquidity is commonly defined as the ease of buying or selling the asset in a market with no effect on its price. From macreconomic view, the most liquid asset is cash, hence a market for any risky asset is ascribed with imperfect liquidity. It is clear that imperfect liquidity hinders both trading and asset pricing, thus it is crucial to retrieve the market rules that can further deteriorate this fragile liquidity. On the other hand, alternative lines of research take the level of liquidity as given and inspect its relation to other market characteristics.

In double auction markets the succinct concept of liquidity is normally partitioned into five main criteria: (i) immediacy, (ii) tightness, (iii) depth, (iv) order book resilience, and (v) and market breadth. Immediacy is a characteristic that refers to the speed of order execution and settlement and, thereby, reflects the overall efficiency of market clearing. Transaction costs, together with any implicit costs, indicate relative tightness of a market. Resilience refers to the market capability to replenish liquidity consumed by aggressive trades and balance appropriately buy and sell side volumes. Market depth is usually measured by the total demand and supply recorded in a limit order book. Lastly, market breadth assesses the density of orders stored in a book, so that a narrow market breadth implies a sparse limit order book with few orders and modest depth. All these characteristics have clear intercepts, hence they jointly determine the ease of converting an asset into cash and vice versa, in other words, asset liquidity.

In my work I take different approaches to evaluate liquidity. Evidently there is no unique measure that gauges all five of its interconnected components. In Chapter 2, I propose an aggregate liquidity factor that is proxied by delay in limit order execution and is based on the notion of immediacy. Under these assumptions I deduce implications of imperfect liquidity on market tightness. In the real option framework proposed in Chapter 3, I model liquidity simply by order flow intensity. I also monitor how different, less abstract market specifications affect this liquidity variable using simulation analysis in Chapter 4. Another statistic that I consider is a price impact that is ingenuously linked to the resilience of a limit order book. Third, I measure the liquidity provision propensity as the ratio of the limit orders in the overall order flow. This characteristic is related both to non-transitory depth and breadth of markets.

1.1.3 Reduced Transparency

Market transparency refers to the ability of market participants to observe non-confidential information about outstanding orders – pre-trade transparency, and executed trades – post-trade transparency. Many modern electronic markets permit iceberg and hidden orders aiming to attract additional liquidity. Each iceberg order has a fixed limit price, a disclosed quantity – total amount of shares to buy or sell, and a peak – the order size publicly displayed in the electronic trading book. Once a peak of an iceberg order is executed, the order is renewed by the same amount and the remaining quantity is reduced accordingly. In this manner iceberg orders assist exposure control and retain price priority with a sacrifice in time priority. If the order size is completely omitted, the order is called hidden. The latter can hide behind other iceberg and limit orders recorded in the book, or have a price distinct to any existing visible passive orders. Therefore, iceberg orders add depth to displayed orders and, as a result, create two market layers: a visible volume and a shadow volume, that is temporarily invisible and mimics the former. Exchange and alternative matching systems with this structure belong to the category of opaque. One concern associated with the promotion of iceberg and hidden order types lies in the discriminative nature of market access, particularly in the extreme case of dark pools. More importantly, hidden orders can distort the price discovery process by exacerbating information asymmetry.

The effect of reduced transparency on market quality is contingent upon the motivation and aggressiveness of traders who rely on iceberg orders: according to empirical evidence, this impact varies from market to market, while theoretical works on this subject are still scarce. Therefore, it is often implicitly assumed that the opportunity to hide granted by opaque markets lures traders because this valuable option diminishes the price impact of their trades. In this thesis I challenge the attractiveness of opaque markets by virtue of an optimal entry model in Chapter 3 and of a dynamic environment with interacting agents in Chapter 4.

1.1.4 Agent-Based Models in Microstructure

Agent-based modelling has proven particularly convenient for exploring the microstructural properties of financial markets. Unlike many theoretical models that often achieve substantive results at the expense of stern simplifying assumptions, agent-based models embrace the sophistication of the trading process and heterogeneity of market participants. One of the first examples that illustrated the benefits of agent-based simulation techniques in application to finance was the Santa Fe Institute artificial stock trading platform created in the late 1980's and early 1990's.

The level of complexity in electronic double auction markets is constantly rising. In recent decades, especially with the development and cheapening of computer technologies, there has been a surge in algorithmic trading and computer-based trading in general. In some cases the use of algorithms implies devising elaborate trading principles and strategies, whereas, when the speed of information processing is a priority, coding bluntly replicates basic rules of thumb. However, the mere diversity of these strategies coupled with the multi-directional nature and, sometimes, high frequency of interactions between various counterparties abrogates this seeming simplicity. Within the complex financial vehicles, as electronic markets undoubtedly are, the emergent institutional impact of trading cannot be deduced by aggregation of individual decisions and actions. To make matters worse, interacting agents usually exhibit bounded rationality. These factors combined challenge the validity of analytical approaches in application to double auction markets. While traditional models appear quite limiting, agent-based models have the capacity to incorporate complexity, and, therefore, gain deeper understanding about the collective behaviour of autonomous agents.

In the meantime, an agent-based simulation approach, like any theoretical framework, enables certain filtering of the modelled complexity level by including some real market principles and omitting others. In fact, it is not even necessary to model the market with substantial precision to reproduce many of its important properties, as proven by previous research. In contrast with empirical studies, this abstraction welcomes to set up more pure experiments within agent-based environment and to test the market impact of new policies *ceteris paribus*. Moreover, agent-based models are especially suitable for exploring non-equilibrium tendencies.

These advantages motivated my analysis of pre-trade transparency regimes in a double auction market via agent-based simulations in Chapter 4. In addition to stochastic models of the optimal behaviour for a single agent in Chapters 2 and 3, the comparative study of interactions among traders and market properties that arise from distinct transparency regimes provides problem interpretation from a different angle.

1.2 Thesis Outline

This thesis belongs to the realm of market microstructure theory, an area of financial economics which examines how the trading behaviour of heterogeneous investors affects the evolution of prices in financial markets. In the summary of ideas introduced above, this thesis contemplates the implications of two market imperfection types: reduced pre-trade transparency and deviations from absolute liquidity. In investigating these problems, I implement stochastic calculus and real options methods alongside agent-based simulations. Each of the three core chapters takes an independent approach to examine the repercussions of weak market liquidity and transparency. Any crossovers in terminology and concepts introduced in each chapter, aside from common definitions accepted in the literature, are noted separately.

Chapter 2 approaches the issue of limited liquidity through the optimal order placement problem of a risk-averse trader in a continuous time framework and introduce a random delay parameter, which defers limit order execution and characterises market liquidity. This framework demonstrates that imperfect liquidity explains order clustering in the proximity of best quotes and the existence of the bid-ask spread. The distribution of expected time-to-fill of limit orders conforms to the empirically observed distribution of trading times, and its variance decreases with liquidity. Finally, two more stylised facts are rationalised in this model: the equilibrium bid-ask spread decreases with liquidity, but increases with agents' risk aversion.

Chapter 3 adjoins the few theoretical attempts in the literature to challenge traders' incentives to participate in opaque trading environments. Through a real option approach I investigate the impact of various market parameters on the optimal order placement time of a risk-neutral trader in a market with hidden liquidity. The model justifies how market opacity can encourage liquidity provision, which, in turn, supports the empirical evidence on the expansion of such trading venues. I demonstrate that transparency in conjunction with liquidity, measured by order flow intensity, determine traders' eagerness to supply liquidity to the opaque market, and that it can be optimal to postpone liquidity provision for later if the order flow is too weak. Moreover, once the trader enters the opaque market, he commits to trade rather quickly relative to his time horizon. Furthermore, error analysis reveals that impatient traders are highly likely to pass over the favourable trade execution offered in the opaque market precisely because of imperfect clarity of information signals, while a prior optimistic bias prompts the trader to engage sooner.

As a complement to static inference on individual level, Chapter 4 presents a study of various transparency regimes and liquidity properties through an artificial limit order market with heterogeneous traders. I establish a model of an artificial market with heterogeneous trading rules and distinct transparency regimes that replicates the long memory properties and empirical patterns in conditional order flow distributions. The results of my experiments suggest that full quote transparency incurs substantial transaction costs for traders and dampens trading activity in an order-driven market, and that exogenous restriction of displayed depth up to several quotes does not substantially alter market performance. The core implication of this model is that the endogenous restriction of displayed quote depth by means of iceberg orders improves market quality in multiple dimensions: it alleviates the problem of adverse selection to patient limit order traders, relieves the average transaction costs, maintains higher liquidity and faster execution, balances the limit order book, and enhances price discovery.

Chapter 5 concludes and summarises the findings obtained from the three methodologies devised in my study. Finally, I discuss briefly potential directions for my future work and propose several extensions that would facilitate further insight into the topics of transparency limitations and liquidity.

1.3 Key Contributions

My contribution to the microstructure literature is at least threefold.

First, I propose a simple stylised order placement model with a synthetic liquidity factor that retrieves a link between liquidity and the bid-ask spread in a limit order market. In addition, this model traces how imperfect liquidity generates the patterns in limit order submissions which are frequently observed in real markets.

Second, I implement a real option approach and show that transparency in conjunction with liquidity determine traders' eagerness to supply liquidity to the opaque market. At the same time, this framework illustrates that market microstructure can benefit from incorporating real option method, which is not broadly applied in this research area at present.

Third, I examine the desirability of opacity through an agent-based model, which is an ample device that complements more rigid theoretical models and empirical research. In line with plentiful empirical observations, this model suggests that opaque markets fulfil its mission to augment liquidity and enhance aggregate market quality, which is a particularly relevant outcome for market transparency regulation.

Chapter 2

Optimal Trading and the Bid-Ask Spread in a Limit Order Market with Imperfect Liquidity

2.1 Introduction

Optimal order placement is a recurrent theme in market microstructure since the solution to this problem ultimately determines market shape and performance. From an investor's standpoint, optimal order placement is the core of successful investment strategy implementation, since skillful trade execution reduces associated transaction costs and amplifies expected returns. Traders construct their submission strategies to benefit from particular market properties and order types, hence the architecture of the market defines their expectations about future price dynamics and trade execution efficiency. In pure quote-driven dealer markets small orders typically execute at the best opposing dealer quote regardless of the order type. In public limit order books, market orders may encounter price improvement, whereas limit orders execution is conditional upon the location where traders place their limit prices relative to the prevailing bid and ask. Market orders trade at the best price currently available at the market and are filled instantaneously. Hence, the actual execution price of a market order is subject to the current market situation. This aspect of trading is known as execution price uncertainty. Limit orders are instructions to trade at the best price available but only if it is not worse than the limit price specified by a trader. The probability that a particular order will be filled depends on its limit price. There exist two main risks associated with limit order trading: execution uncertainty, when the market moves away from the submitted limit price and hence the agent never trades, and ex post regret, when for various reasons prices move towards and through the limit price. The goals of traders operating in a competitive environment of non-intermediated double auction markets are distinct from those of market makers. The fundamental distinction between the models of dealer quoting and a generic trader problem is that the former is essentially indifferent to execution and his objective is to set a spread to control his inventory position.[1] In limit order markets agents have multiple reasons to trade and various financial instruments available to them to execute a particular trade. Primarily, though, each trader entering a non-intermediated market must decide upon the type of order to use: either a market order or a limit order.

Substantial empirical evidence suggests that limit orders play a dominant role in financial markets. For instance, Harris and Hasbrouck (1996) find that limit orders generally perform best even in the presence of a non-execution penalty and market price improvement. According to Guéant et al. (2012), more than half of all trades, approximately 60%, are obtained passively, that is, orders fill the queue to trade rather than consume liquidity. Biais et al. (1995) provide an empirical analysis of the order flow of the Paris Bourse, which represents a pure limit order market. They find that traders' strategies vary with market conditions, with orders submitted inside the quotes more frequently at times when spreads are wide. The motivation to choose a limit order over a market order in real markets is obfuscated by various subtle effects, for example: the discrepancy in transaction fees,[2] the option to submit multiple orders simultaneously, the existence of several distinct markets for the same asset, and the possibility to withdraw the order. Some of previous theoretical studies are based on the division of traders into two main groups: liquidity suppliers that trade with limit orders, and liquidity demanders that have stronger immediacy preferences (see, for example, the articles of Foucault et al. (2005); Harris (1998); Rosu (2009)). In contrast, other frameworks, including Handa and Schwartz (1996); Parlour (1998) and Foucault (1999) to mention a few early works, endogenise the choice between market and limit order placement. In this chapter I expand the latter part of the literature and develop a framework where depending on the current market state the trader sets out either to provide or to consume liquidity.

In practice, even though investment and trading decisions are formulated jointly, they are usually analysed and accomplished somewhat in isolation. The present model reflects precisely such allocation of tasks since it does not take into account the portfolio the trader holds and how the outcome of his trading operations affects the balance and the value of his portfolio. Instead, the model assumes that the agent enters the market with a given trading goal and his task is to devise an optimal execution strategy given market characteristics at the time of his arrival. The set of information available to traders is an important issue in this context. There is abundant empirical evidence in the market microstructure literature suggesting that information asymmetries between agents shape market dynamics and that price movements are often triggered by information updates.

[1] The dealers choose optimally the spread that maximises their expected gains from trading with uninformed liquidity traders net of expected losses incurred by trading with informed.

[2] Maker/taker market microstructure, for instance, encourages liquidity provision by offering a fee discount when a limit order is submitted and penalises liquidity consumers by higher charges.

A number of early studies showed that the information component of the bid-ask spread is slender; according to Huang and Stoll (1997), on average it compounds less than 12% of the spread. Yet, more contemporary findings indicate that the information component is significant and amounts for as much as 80% of the spread (Gould et al., 2010). Information asymmetry is usually defined in a fairly broad sense and it does not necessarily imply, for instance, any form of legal or illegal insider trading. Thus, simply because the tools through which market participants assess the market vary, they draw different predictions from the same market conditions. However, superior or rather heterogeneous information is not the only basis for trading, as was demonstrated by Milgrom and Stockey (1982). On this basis, information effects per se are excluded from the present analysis. The model incorporates traders' individual expectations of the future asset price, while leaving the grounds for this valuation beyond the scope of the present study.

In this chapter I consider the problem of a trader who has to liquidate a position in an actively traded asset within a given period of time and forms his strategy based upon the market dynamics represented by the bid and ask prices. Building upon the framework of Iori et al. (2003), I provide a conceptual improvement by incorporating an exogenous limit order execution factor – an exponential random delay. Moreover, I derive a static solution to the limit order trading problem for the quadratic utility preferences and examine how different risk perceptions affect the optimal strategy of a trader in a limit order market and his waiting time. This formulation proves more advantageous both in terms of interpretation and the ease of potential calibration to data. Furthermore, I explain the non-transient bid-ask spread and identify the determinants of its width in equilibrium.

The remainder of this chapter proceeds as follows. Section 2.2 reviews relevant theoretical and empirical literature. Section 2.3 describes the market in which traders submit their orders, outlines the clearing mechanism and formulates the problem of a risk-averse trader operating in this market. In Section 2.4 I look at the distribution of limit order trading times implied by this market design. Subsequent Section 2.5 and Section 2.6 explore the properties of the optimal strategy and two special cases respectively. Section 2.7 reports comparative statics analysis for the key model drivers. Finally, the existence of equilibrium spread and appropriate conditions are discussed in Section 2.8. The chapter concludes in Section 2.9 with a brief summary and suggestions for further research.

2.2 Literature Review

There is an extensive literature on the subject of optimal order submission strategies in limit order markets. The main distinction between the theoretical approaches adopted in various studies lies in the definition of a limit order execution mechanism and, consequently, the resulting probability distribution.

Equilibrium analysis of order-driven markets has been realised by Chakravarty and Holden (1995); Foucault (1999); Foucault et al. (2005); Goettler et al. (2005); Kumar and Seppi (1994); Parlour (1998) and Rosu (2009). All of these models are variants of a dynamic multi-agent sequential bargaining game where heterogeneous traders derive their best response order submission strategies. Parlour (1998) starts from the premise that the probability of execution of a sell limit order depends on the arrival of buy market orders and that the relative attractiveness of buy market orders depends on the relative attractiveness of buy limit orders, and concludes that execution probabilities of buy and sell limit orders are determined jointly over time. In situations when prices are fixed, which holds in equilibrium, this framework suggests that optimal order placement is contingent upon a single factor – the distribution of agents' impatience characteristic. Allowing for market fluctuations, Foucault (1999) describes the asset price via a binomial model and assumes that limit orders are valid only for one period, therefore, at each point in time the book is either full or empty. In this setting the probability of a limit order execution is endogenous and part of the execution risk arises from the next trader's order type. The focus on equilibrium analysis of the optimal behaviour yields numerous implications that are coherent with the documented market observations, and these models have proved especially useful for policy-makers. However, the non-synchronous nature of trading in real markets challenges this approach: while in equilibrium all market participants should get zero profits, the depth of a real limit order book is usually insufficient to drive average expected profits to zero.

A separate branch of optimal order placement literature, where individual order submissions are encapsulated in the asset price dynamics, was initiated by Cohen et al. (1981). In their seminal paper, Cohen et al. (1981) propel the theoretical analysis of the optimal choice between market and limit orders and build a framework where the probability of order execution is contingent upon future price movements and associated probability densities. Trading takes place when the trajectory of the best quote crosses the barrier determined by the limit price for the first time. Cohen et al. (1981) model the underlying security price with a compound Poisson process, which, by very definition, invokes a jump in the probability of execution: if a time-constrained trader is willing to buy a stock via a limit order and chooses a limit price infinitely close to the current best ask, his probability of trading never attains unity. This property permitted to establish a so-called *gravitational pull effect*: when the bid-ask spread is narrow, the benefit of a price improvement with a limit order becomes small compared to the risk of non-execution, so traders are pulled to use market orders instead. Consequently, Cohen et al. (1981) argue that a limit order strategy is not always superior to trading with market orders and that refraining from submitting any order might even be the most sensible option. This model adequately captures the trade-off between a favourable price and a higher order execution probability and, in this sense, draws a line between market and limit orders.[3] Cohen et al. (1981) further demonstrate that as the order arrival rate increases the Poisson process converges to a Wiener process, eliminating the discontinuity in the execution probability function. The model setup, however, is too complex to obtain a closed-form solution and their analysis remains mainly qualitative.

Langnau and Punchev (2011) concentrate deliberately on the issue of adequate price modeling in a public limit order book. Adopting the results of Kou and Wang (2003), they compare the implications of a pure diffusion and a double exponential jump diffusion (DEJD) mid-point price specifications, with the appealing memorylessness property and leptokurtic returns in the latter case. They arrive at a closed-form solution for the first passage time with distinct expressions for buy and sell strategies in the DEJD case. As a main finding, Langnau and Punchev (2011) establish that the DEJD price specification accommodates the asymmetric shape of a limit order book as well as a fat-tailed distribution of log-returns and is compatible with equilibrium market conditions, whereas the log-normal price specification is not. However, Langnau and Punchev (2011) do not address directly the optimal trading problem. Although DEJD prices complicate parameter calibration, it is potentially beneficial from the tractability viewpoint and I reserve this for future research.

Given the analytical complexity of any setting with jumps in prices, it seems reasonable to examine the problem of optimal strategy within the context of a continuous time diffusion price. There are several examples of this approach in the financial and econometric literature. For instance, Iori et al. (2003) show that despite its obvious shortcomings, log-normal prices still replicate optimal limit order strategies that are coherent with traders' behaviour observed in financial market. A mean-reverting price specification, particularly relevant for commodities, depicts an interesting cross-over effect: the optimal value of the strategy increases with the speed of reversion for urgent expiry times, while decreases for longer trading horizons. Nevertheless, pure first passage time models such as this cannot justify the existence of the spread due to inability to differentiate between a marketable limit order and a market order, or more precisely, to distinguish between the time-to-first-fill and completion time of a trade.

An attempt to preserve the appealing mathematical lightness of first passage time models and bridge it with the notion of imperfect liquidity was made by Harris (1998). Harris (1998) improves the framework with pure diffusion prices by adding a supplemental criterion – an aggregated factor of the degree of execution difficulty. The degree of difficulty in limit order execution is defined as an additional barrier which a limit price has to pass before a limit order is filled. Therefore, it is not sufficient to become the best price on the same side of the market, a limit order has to supersede this best price: for instance, a sell limit order is executed when the submitted limit order price is lower than the best ask less the difficulty parameter. Two execution mechanisms were juxtaposed:

[3] Angel (1994) uses a similar concept to model limit order submissions in a market where order arrival is described by a standard Poisson process.

with certainty and with some probability. Using a comparative statics analysis based on a numerical solution, Harris (1998) confirms that the probability of order execution depends positively on this difficulty factor for both schemes. An alternative stylised interpretation of the problem is given by Hasbrouck (2006). Assuming that traders on the opposite side of the book are ascribed with unobserved reservation prices, a random collateral barrier that follows an exponential distribution is analysed. Hasbrouck (2006) argues that the number of potential counterparties decreases as the distribution of reservation prices becomes more skewed, making the optimal strategy more aggressive. Although very valid, the theoretical approaches presented by Harris (1998) and Hasbrouck (2006) have certain limitations. Disentangling the impacts of these external factors, i.e. distribution of reservation prices or the degree of execution difficulty, from other determinants of order aggressiveness and actual trading times poses a serious inference challenge. Instead, I introduce a tractable parameter to characterise limit orders which can actually be estimated from observable data.[4]

2.3 The Model

Consider the problem of an investor who has a position in a traded asset which has to be liquidated within a pre-specified time horizon. One option the agent has is to use a market order and trade at the best available price at once. Alternatively, he can submit a limit order and hope to trade at a more favorable price. In a double auction market a transaction occurs when a market order hits the quote on the opposite side of the book. Assume that there is no information asymmetry and future price dynamics depend only upon public information. Without loss of generality, the ensuing analysis focuses on the problem of a seller.

A trader is alloted the task of selling one unit of an asset and has to complete the trade by deadline T. In order to optimise the price he is willing to adopt a limit order strategy and takes into account a penalty for the non-execution event. The current price of the security is determined by the best bid b_0, the highest buying price, and the best ask a_0, the lowest selling price in the market. Choosing to trade at the market, the agent receives an immediate profit. If the agent adopts a limit order strategy, he places an order at his preferred limit ask price K_a. Assume that the trader cannot revise his limit order before expiry, however, it is possible to convert to a market order at maturity if his limit order was not filled. Therefore, if his limit order is not executed within the horizon T, the agent has to sell the asset at the best available price that guarantees immediate trading.[5]

Imperfect liquidity is introduced by a random delay in execution. When a trader submits a sell

[4] For instance, in 2005 the SEC adopted Rule 605 requiring all market centers to disclose certain order execution information, thus facilitating post-trade transparency. This regulation requires, among other things, to make publicly available information on the order execution speed.

[5] The assumption that order size is one unit ensures that the market will absorb the trade. However, one should bear in mind that this does not necessarily hold for large orders and switching to market orders at maturity might inflict significant price discounts.

limit order at K_a to the book, he is competing with other potential sellers, or the best ask process a_t. Placing a limit order far from the current quotes implies a more intense competition and increases the chances that the opportunity to trade will not arise before expiry time. Limit orders in the book are executed according to a first-in first-out protocol. However, once the limit order of an agent becomes the best price in the market, a random execution delay ensues. The primary source of delay comes from the possibility that another trader places an order ahead of the trader in question. An impatient trader can arrive and put a more aggressive limit order at a marginally lower price. If this happens, the patient trader loses the price priority while retaining the time priority at his price. Unless there is a fundamental market shock, this price distress should quickly recover and his order will trade soon. In other words, the delay indicates the time it will take for the price to return to the trend. Also, the delay in trading may happen due to the fact that somebody else might have put an order at exactly the same price but earlier than the agent in question. This is a salient feature of markets with hidden liquidity, since traders normally have no information about invisible quote depth. As a result, even when placing an order at a given price traders have no capability to assess how long it will take to fill hidden orders ahead of them. In this model the delay is usually insignificant relative to the trader's timescale, but there is a small probability that the delay will be sufficiently long and hence will considerably distort the schedule of the agent's trading. Therefore, in this market the transaction occurs with an unforeseen delay ε which is independent of the asset prices and is sampled from an exponential distribution with constant intensity λ, that is $\varepsilon \sim \exp(\lambda)$. Clearly, in most cases, once the market is trading close enough to the quote that the agent has previously submitted, it will not move away swiftly. This market design implies, as will be shown later, that a non-transient bid-ask spread arises from the costs of waiting.

In a situation with a random delay the agent cannot bear additional risk at maturity, hence he is forced to use a market order instead and sell the security at b_T. Submitting a limit order at at any price greater than the best bid is not a sure trade and given that the distribution of the random delay is equivalent for any price, the trader has no incentive to place the order inside the spread, i.e. below a_0. This strategy yields the following expression for the discounted terminal payoff:

$$V_\lambda(K_a; \vec{v}) = e^{-\delta(\tau+\varepsilon)} K_a I_{\{\tau+\varepsilon\leq T\}} + e^{-\delta T} b_T I_{\{\tau+\varepsilon>T\}}, \tag{2.1}$$

with $\tau = \inf\{t \geq 0 : \ a_t = K_a\}$ and $\varepsilon \sim \exp(\lambda)$.

Numerous studies solve for the optimal strategy from the standpoint of a risk-neutral agent. However, if risks cannot be hedged away, the trader is concerned not only with expected payoffs but also with the range where future payoffs might lay; thus, he should account for the variance in the future wealth. The trader with risk aversion coefficient φ determines his optimal limit price K_a^*

by maximising the mean-variance utility:

$$EU_\lambda(K_a; \vec{v}, \varphi) \equiv \max_{K_a \geq a_0} \left\{ E\left[V_\lambda(K_a; \vec{v})\right] - \varphi \cdot Var\left[V_\lambda(K_a; \vec{v})\right] \right\}. \tag{2.2}$$

The final decision of a trader is formed through comparison of b_0, the price at which he can close the position instantaneously, and the maximum expected mean-variance utility $EU_\lambda(K_a^*; \vec{v}, \varphi)$ that can be attained via a limit order at the optimal price K_a^*.

Despite the presence of a delay, this market is sufficiently liquid and the price dynamics are described by a continuous process. A pair of stochastic log-normal processes b_t and a_t describe the trajectories of the bid and ask prices in the book respectively:[6]

$$db_t = \mu_b b_t dt + \sigma_b b_t dW_t^b, \tag{2.3}$$

$$da_t = \mu_a a_t dt + \sigma_a a_t dW_t^a, \tag{2.4}$$

where W_t^a and W_t^b are ρ-correlated standard Brownian motions, $E\left[dW_t^a \, dW_t^b\right] = \rho dt$. This framework is primarily concerned with short-term decisions when market conditions do not change substantially, so the assumption of deterministic price trends is viable.[7]

2.4 The Distribution of Trading Times

This section examines the properties of waiting times implied by market dynamics and the trader's submissions. Define the time-to-fill of a limit order as $\theta = \tau + \varepsilon$, where τ is the time it takes to become the best price and ε is a random delay in order execution. The probability density function of the time-to-fill, as depicted in Figure 2.1, exhibits a negative exponential shape with a peak.[8] As the intensity rate λ increases, the peak becomes more pronounced, implying that faster order execution is more likely other things equal. In other words, higher liquidity reduces the variance of the time-to-fill.

The relationship between the distribution of trading time in a market without delay and in a market with a random delay is stated in the following proposition.

Proposition 2.4.1. *Let $P(\tau \leq t)$ be the cumulative distribution of limit order time-to-fill in a perfectly liquid market, then the cumulative distribution function of waiting times in a market with*

[6] The best quotes contain sufficiently rich information to form a trading strategy. Kozhan and Salmon (2012), for instance, show by means of an FX market that little value is extracted from the data beyond the best quotes, especially with the advent of algorithmic trading.

[7] The evaluation of the spread non-negativity conditions is given in Appendix D.

[8] The absence of a peak near zero in the results obtained by Lo et al. (2002) can be partially attributed to the discrete nature of the data, whereas the result of the model given in this chapter applies to continuous time.

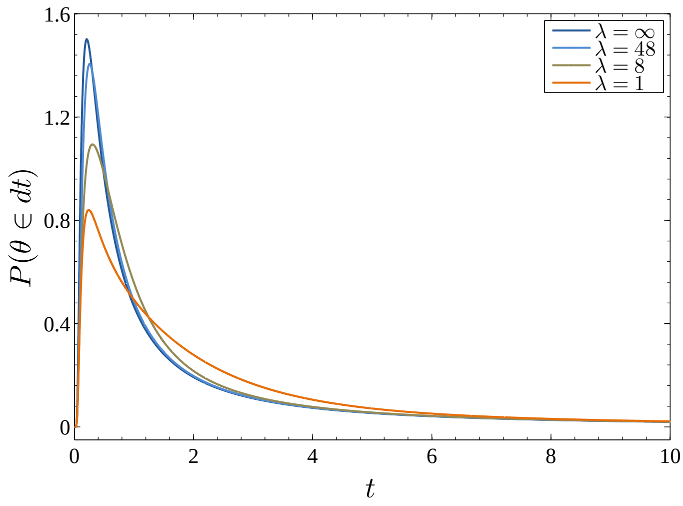


Figure 2.1: The probability density function of the time-to-fill of a sell limit order at $K_a/a_0 = 1.01$. Results are shown for parameters in Table 2.2.

moderate delays $P(\theta \leq t)$ is approximated by

$$P(\theta \leq t) = P(\tau \leq t) - \frac{x/\lambda}{\sigma_a t\sqrt{t}}\, n\left(\frac{x - A_1\sigma_a^2 t}{\sigma_a\sqrt{t}}\right). \tag{2.5}$$

The probability density function of the time-to-fill in a market with moderate delays $P(\theta \in dt)$ is proportional to the probability density of the time-to-fill in a perfectly liquid market $P(\tau \in dt)$:

$$P(\theta \in dt) = P(\tau \in dt)\left[1 - \frac{(x^2 - A_1^2\sigma_a^4 t^2 - 3t)/\lambda}{2\sigma_a^2 t^2}\right], \tag{2.6}$$

where $x = \ln\left(\frac{K_a}{a_0}\right)$ and $A_1 = \frac{\mu_a - \sigma_a^2/2}{\sigma_a^2}$.

Proof. See Appendix A. □

Proposition 2.4.1 complies with the intuition that the difference in limit order execution is most sensitive to market liquidity when medium-length maturities are concerned, since both equations (2.5) and (2.6) pertain a non-monotonic dependence on time t.

Further, the statistics in Table 2.1 reflect the impact of a random delay on the time-to-fill. Taking into account that delays are independent of prices, the mean time-to-fill is calculated as $E[\theta] = E[\tau] + E[\varepsilon]$. As Figure 2.2(a) reveals, the average waiting time $\bar{\theta}$ grows linearly with the distance-to-fill of a limit order. However, the contribution of the market liquidity parameter λ is very small and there is no marked difference in the submitted limit price. Given that the distribution of the time-to-fill is considerably skewed, its mean value is not an informative statistic. Figure 2.2(b) shows the mode time-to-fill $\hat{\theta}$, that is the most common value, for a range of admissible limit prices. For

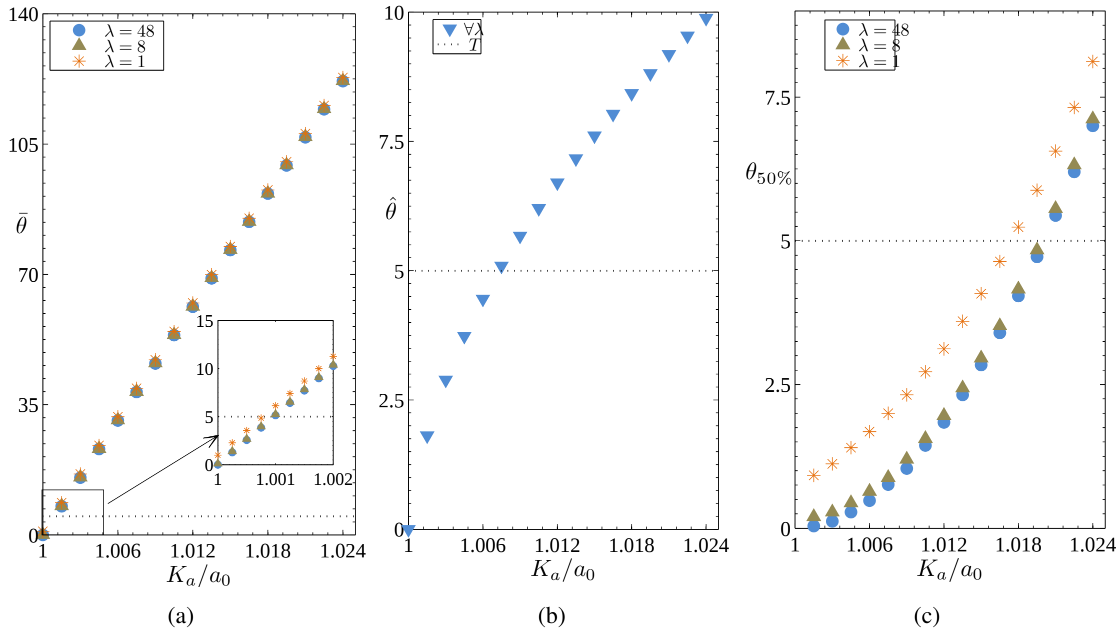


Figure 2.2: The characteristics of the most common time-to-fill of sell limit orders submitted at different prices $K_a \geq a_0$: (a) the average time-to-fill, (b) the mode time-to-fill, (c) the median time-to-fill. Results are shown for parameters in Table 2.2.

any level of liquidity, the more aggressive a limit order strategy, the shorter is the mode execution time. The mode of the sum of independent variables is the sum of their modes and since the mode of an exponentially distributed random variable is zero, the framework does not distinguish between the most common time-to-fill $\hat{\theta}$ and the most common first passage time $\hat{\tau}$. Further, I plot in Figure 2.2(c) the median of the time-to-fill $\theta_{50\%}$, which indicates the waiting time before limit order execution that occurs with 50% probability. I observe, for instance, that if a trader submits a sell order at 2% above the ask in an almost perfectly liquid market ($\lambda = 48$), then he is equally likely to transact earlier or later than his horizon of five days. In a market with distorted liquidity ($\lambda = 1$) the trader must use a more aggressive order at 1.7% above time zero best ask to achieve this. As Figure 2.2(c) demonstrates, the median trading time increases as orders become more passive, and the higher the liquidity, the shorter the median time-to-fill. One possible interpretation of these results is that imperfect market liquidity requires more patience from traders.

In their empirical study based on survival analysis of the institutional brokerage limit-order dataset Lo et al. (2002) observe exponentially distributed trading times. Cho and Nelling (2000) suggest that given market orders arrive as a non-homogeneous Poisson process the execution times of limit orders follow a Weibull distribution. The estimations from a duration model based on Cho and Nelling (2000) provide a good fit of empirical observations by an exponential probability density as well as Weibull. It is noteworthy then that my theoretical result is compatible with a

Table 2.1: The average descriptive statistics of the execution delay and the limit order time-to-first.

Statistic	first passage time	delay	time-to-fill
mean	$\bar{\tau} = \frac{\ln\left(\frac{K_a}{a_0}\right)}{\mu_a - \sigma_a^2/2}$	$\bar{\varepsilon} = \frac{1}{\lambda}$	$\bar{\theta} = \frac{\ln\left(\frac{K_a}{a_0}\right)}{\mu_a - \sigma_a^2/2} + \frac{1}{\lambda}$
mode	$\hat{\tau} = \frac{\sqrt{4\ln\left(\frac{K_a}{a_0}\right)^2 (\mu_a - \sigma_a^2/2)^2 + 9\sigma_a^4} - 3\sigma_a^2}{2(\mu_a - \sigma_a^2/2)^2}$	$\hat{\varepsilon} = 0$	$\hat{\theta} = \frac{\sqrt{4\ln\left(\frac{K_a}{a_0}\right)^2 (\mu_a - \sigma_a^2/2)^2 + 9\sigma_a^4} - 3\sigma_a^2}{2(\mu_a - \sigma_a^2/2)^2}$
median	$\tau_{50\%}: \quad P(\tau < \tau_{50\%}) = \frac{1}{2}$	$\varepsilon_{50\%} = \frac{\ln 2}{\lambda}$	$\theta_{50\%}: \quad P(\theta < \theta_{50\%}) = \frac{1}{2}$

Weibull distribution specification for a shape parameter smaller or equal to unity.

Empirical research also reveals that there is a correlation between execution time and limit order prices and the causality of this relationship is bilateral. Tkatch and Kandel (2006) find a significant causal impact of expected execution time on investors' decisions regarding the order type choice. Lo et al. (2002) reveal that limit order execution times increase as limit prices become more passive and move further away from the quotes, which is partially an outcome of the price priority rule. According to my market setup, the expected time-to-fill is longer for sell limit orders at higher limit sell prices. Moreover, it captures how changes in liquidity levels alter the distribution of limit order execution times: median times-to-fill prolong in a market with delays in contrast to the perfect liquidity case. This pattern echoes the findings of Lo et al. (2002), who estimate the cumulative probability densities of the actual limit order execution times and compare it to their hypothetical counterpart calculated as the first hitting times of geometric Brownian motion. The latter appears to understate largely expected trading times. Lo et al. (2002) extensively discuss the histograms of empirical time-to-execution for limit orders, which exhibit exponential distribution and differ from the analytical benchmark case not only in one or two moments but over their entire support. Therefore, without analysing directly how order placement of a certain trader affects the market, the model with a random delay in limit order execution grasps the statistical properties of trading times and relates the expected time-to-fill to order aggressiveness. Overall, the timescale immanent to this framework conforms to the known empirical findings.

2.5 Optimal Strategy

The risk-averse trader formulates his optimal strategy in two steps: (i) choose the optimal limit price K_a^*, and (ii) compare the expected utility at K_a^* to a market order at b_0. In other words, the trader goes with the strategy which yields the highest expected utility. The following proposition provides the power function of a limit order payoff.

Proposition 2.5.1. *Let the market bid and ask prices follow positively correlated log-normal stochas-*

tic processes (2.3) and (2.4). Denote the power function of the profit from selling a unit of security via a limit order by $G(K_a;\vec{v},\gamma) \equiv [V_\lambda(K_a;\vec{v})]^\gamma$. *The expected value of* $EG(K_a;\vec{v},\gamma)$ *for a limit price* $K_a \geq a_0$ *in a market with a random delay in order execution* $\varepsilon \sim \exp(\lambda)$ *is given by*[9]

$$\begin{aligned} EG(K_a;\vec{v},\gamma) &= K_a^\gamma \frac{\lambda}{\lambda+\gamma\delta}\Bigg[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x-A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)+\left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x+A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) \\ &- e^{-(\lambda+\gamma\delta)T}\left(\left(\frac{K_a}{a_0}\right)^{A_1+A_3} N\left(\frac{-x-A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)+\left(\frac{K_a}{a_0}\right)^{A_1-A_3} N\left(\frac{-x+A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right)\Bigg] \\ &+ b_0^\gamma\, e^{\gamma\left(\mu_b+(\gamma-1)\sigma_b^2/2-\delta\right)T}\Bigg[N\left(\frac{x-A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)-\left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x-A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) \\ &+ e^{-\lambda T}\left(\left(\frac{K_a}{a_0}\right)^{A_4+A_5} N\left(\frac{-x-A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)+\left(\frac{K_a}{a_0}\right)^{A_4-A_5} N\left(\frac{-x+A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right)\Bigg], \end{aligned} \tag{2.7}$$

where $x = \ln\left(\frac{K_a}{a_0}\right)$ *and the constants* $A_1 = \frac{\mu_a-\sigma_a^2/2}{\sigma_a^2}$, $A_2 = \frac{\sqrt{(\mu_a-\sigma_a^2/2)^2+2\gamma\delta\sigma_a^2}}{\sigma_a^2}$, $A_3 = \frac{\sqrt{(\mu_a-\sigma_a^2/2)^2-2\lambda\sigma_a^2}}{\sigma_a^2}$, $A_4 = \frac{\mu_a-\sigma_a^2/2+\gamma\rho\sigma_a\sigma_b}{\sigma_a^2}$, $A_5 = \frac{\sqrt{(\mu_a-\sigma_a^2/2+\gamma\rho\sigma_a\sigma_b)^2-2\lambda\sigma_a^2}}{\sigma_a^2}$; *with the parameter constraints* $\lambda \in \left[0; \frac{(\mu_a-\sigma_a^2/2)^2}{2\sigma_a^2}\right]$ *and* $\rho \geq 0$.

Proof. See Appendix B. □

Consider the case when a limit order to sell is submitted at the current ask. The expected value of this strategy is

$$EV_\lambda(K_a = a_0;\vec{v}) = a_0\frac{\lambda}{\lambda+\delta}\left(1-e^{-(\lambda+\delta)T}\right)+b_0\, e^{(\mu_b-\delta-\lambda)T}. \tag{2.8}$$

This expression suggests that the longer the mean delay $\bar{\varepsilon} = 1/\lambda$, the smaller the weight attached to the profit from using a limit order at the current ask a_0, the first term, and the larger the contribution of a market order submitted at maturity, the second term. In the extreme case of illiquidity when $\lambda = 0$, that is, when the delay is infinitely long, the expected profit from selling the asset equals the discounted expected best bid at maturity $\lim_{\lambda\to 0} EV_\lambda(K_a = a_0;\vec{v}) = e^{-\delta T}E[b_T]$. In fact, no limit order will trade before the maturity if $\lambda = 0$. More precisely, formula (2.8) reveals that the expected utility does not depend on K_a, instead, it equals exactly the expected utility of a market order at time T:

$$\lim_{\lambda\to 0} EU_\lambda(K_a;\vec{v},\varphi) = b_0\, e^{(\mu_b-\delta)T} - \varphi b_0^2\, e^{2(\mu_b-\delta)T}\left[e^{\sigma_b^2 T}-1\right]. \tag{2.9}$$

Furthermore, an agent prefers a market sell order upon maturity at price b_T per share instead of

[9] Using the first and second moments of the expected profit from selling, the expected utility is calculated as $EU_\lambda(K_a;\vec{v},\varphi) = EG(K_a;\vec{v},1) - \varphi\left(EG(K_a;\vec{v},2) - [EG(K_a;\vec{v},1)]^2\right)$.

submitting a market order immediately at b_0 if his risk-aversion is very lenient and satisfies:

$$\varphi \leq \frac{e^{(\mu_b-\delta)T}-1}{e^{2(\mu_b-\delta)T}b_0\left(e^{\sigma_b^2 T}-1\right)}. \tag{2.10}$$

It follows from here that the trader's patience is guided by the trend on the opposite side of the limit order book: if the drift of the best bid μ_b is high enough, many traders are motivated to provide liquidity by filling the limit order book with orders to sell.

The closed-form representation (2.8) given in Proposition 2.5.1 is valid for $0 \leq \lambda \leq \bar{\lambda}$. This constraint on the delay intensity rate $\bar{\lambda}$ arises from the distribution properties of the first passage time of a_t.[10] However, the maximum value $\bar{\lambda}$ imposes a strong liquidity restriction, as the numerical example in Section 2.7 reminds.[11] I use numerical integration methods to calculate the expected utility and find the optimal limit price for intermediate values of $\bar{\lambda} < \lambda \ll \infty$.

2.6 Special Cases

In this section I highlight a number of limiting cases that are consistently incorporated into the developed framework. In particular, I demonstrate that the model of a limit order market with a random execution delay permits perfect liquidity. Furthermore, two important special cases of this model allow to isolate two types of risks inherent to limit orders: I investigate the range of admissible limit prices chosen by traders when there is no time pressure, and when the volatility of the asset prices approaches zero.

2.6.1 Perfect Liquidity

Infinitely high levels of λ are pertinent to perfectly liquid markets, which simultaneously implies that the first passage time of any limit order placed in the book is equivalent to its time-to-fill. The following proposition provides the valuation formula of a selling strategy via a limit order in the case of perfect liquidity.

Proposition 2.6.1. *Let the market bid and ask prices follow ρ-correlated log-normal stochastic processes (2.3) and (2.4). Denote the power function of the profit from selling a unit of security via a limit order by $G(K_a; \vec{v}, \gamma) \equiv [V_\lambda(K_a; \vec{v})]^\gamma$. The expected value of $EG(K_a; \vec{v}, \gamma)$ for a limit price $K_a \geq a_0$ in a market with a small random delay in order execution $\varepsilon \sim \exp(\lambda)$ converges to the*

[10] As explained in Appendix B, the condition $\lambda < \bar{\lambda}$ is due to the restriction of one of the natural parameters of the inverse Gaussian distribution of the trading time.

[11] The analytical examination of expected utility boundaries for intermediate liquidity levels is carried out in Appendix E.

value

$$\begin{aligned}\lim_{\lambda\to\infty} EG\left(K_a;\vec{v},\gamma\right) \;=\;& K_a^{\gamma}\left[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x-A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)+\left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x+A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right] \\ &+\; b_0^{\gamma}\, e^{\gamma\left(\mu_b+(\gamma-1)\sigma_b^2/2-\delta\right)T}\left[N\left(\frac{x-A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)-\left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x-A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right]\end{aligned} \tag{2.11}$$

where $x = \ln\left(\frac{K_a}{a_0}\right)$ *and* $A_1 = \frac{\mu_a-\sigma_a^2/2}{\sigma_a^2}$, $A_2 = \frac{\sqrt{(\mu_a-\sigma_a^2/2)^2+2\gamma\delta\sigma_a^2}}{\sigma_a^2}$, $A_4 = \frac{\mu_a-\sigma_a^2/2+\gamma\rho\sigma_a\sigma_b}{\sigma_a^2}$.

Proof. See Appendix C. □

This result replicates the valuation formula derived in Iori et al. (2003) for the market where limit orders trade upon achieving the beginning of the queue on the relevant side of the book. Accordingly, in an infinitely liquid market a trader will resolve to a limit order at a_T in the non-execution event rather than pick the best buying order at b_T. In particular, this article explores the properties of optimal order placement in cases of a zero and positive constant spread. My model demonstrates, however, that the setting of Iori et al. (2003), where a limit sell order is executed as soon as it matches the current best ask, actually requires a zero bid-ask spread, i.e. $a_t = b_t,\ \forall t$ in equilibrium, as Section 2.8 emphasises. The work of Langnau and Punchev (2011) supports this conclusion, since the authors prove that there is no spread in a market where prices follow geometric Brownian motion unlike the market with jumps in prices.

2.6.2 Infinite Time Horizon

Consider the case of infinitely long time horizon. Both for illiquid and liquid markets, the power function of selling an asset with a limit order reduces to:[12]

$$\lim_{T\to\infty} EG_\lambda(K_a;\vec{v},\gamma) = \frac{\lambda}{\lambda+\gamma\delta} K_a^{\gamma}\left(\frac{K_a}{a_0}\right)^{A_1-A_2}. \tag{2.12}$$

Notice that there is a monotonic increasing relationship between the limit price K_a and the power function (2.12). This means that both the expected profit and the variance grow with K_a. Once the time pressure is removed, the penalty which incurs paying the spread at maturity simply vanishes for a risk-neutral trader and encourages him to submit K_a as far as possible. However, since variance increases faster than the mean value, for a risk-averse trader the risks of a limit order strategy will outweigh the benefits for high limit prices and he will switch to a more aggressive order placement. Also, the limit given in expression (2.12) is smaller than K_a^{γ}, since $K_a \geq a_0$ and

[12]In case $\lambda \leq \bar{\lambda}$ it follows from the result in Proposition 2.5.1. For the case $\lambda > \bar{\lambda}$ I apply the Laplace integral approximation in Lemma 2.9.1 for fixed λ and $T \to \infty$. The condition that guarantees the convergence is $\gamma(\mu_b + (\gamma - 1)\sigma_b^2/2 - \delta) \leq \lambda$ and it holds for a wide range of parameters.

$A_2 > A_1$ for any values of the parameters, the discrepancy being especially stark when the liquidity λ is low. Hence, for certain parameters the trader will even prefer to use a market order to sell the asset straightaway.

2.6.3 No Price Uncertainty

When the volatility of the underlying security approaches zero, its price drifts at a constant rate per unit of time. Therefore, the maximum that the best ask can attain prior to maturity is known to be $a_T = a_0 e^{\mu_a T}$, and the best bid is $b_T = b_0 e^{\mu_b T}$. The moment when the limit sell order K_a hits the quote is calculated as $\tilde{\tau} = \frac{\ln(K_a/a_0)}{\mu_a}$. In this situation the trader bears no price risk and the only risk he faces is linked to the non-execution of his order due to delays.[13]

$$\lim_{\sigma \to 0} EG_\lambda(K_a; \vec{v}, \gamma) = \frac{\lambda}{\lambda + \gamma\delta} K_a^\gamma \left[\left(\frac{K_a}{a_0} \right)^{-\gamma\delta/\mu_a} - e^{-(\lambda+\gamma\delta)T} \left(\frac{K_a}{a_0} \right)^{\lambda/\mu_a} \right] + b_0^\gamma e^{(\gamma(\mu_b - \delta) - \lambda)T} \left(\frac{K_a}{a_0} \right)^{\lambda/\mu_a}. \tag{2.13}$$

Considering the deterministic nature of prices, the trader will optimise his strategy only over the subset of limit prices from the interval $a_0 \leq K_a \leq a_0 \cdot e^{\mu_a T}$, to ensure that $\tilde{\tau} \leq T$.

2.7 Numerical Example

2.7.1 Baseline Parameters

The values of parameters used in the numerical example are summarised in Table 2.2. I set the time-zero best ask in the book equal to £1000 with the initial spread of £5. The discount factor in this market is 5% per annum with 10% expected drift and 20% annual volatility, and a weak positive correlation in prices. It follows immediately from the assumption of no information asymmetry that all traders are equally informed about the value of the security. The bid and ask prices represent valuation on demand and supply sides of the same asset and should not diverge significantly at any point in time. Given these considerations, I work with the cases when the drifts of bid and ask are the same in order to preserve stationarity of the spread.[14] The time horizon is 5 days in a benchmark case, with the average delay length 1/8 of a trading day,[15] and the risk-aversion parameter 0.01.

Using analytical expressions (2.11) and (2.8) for the expected payoff in a perfectly and imperfectly liquid markets respectively, I find the optimal limit price at which a trader should submit his order to achieve the maximum level of expected mean-variance utility. Further, I implement

[13] $EG_\lambda(K_a; \vec{v}, \gamma) = E\left[K_a^\gamma e^{-\gamma\delta(\tilde{\tau}+\varepsilon)} I_{\{\varepsilon \leq T-\tilde{\tau}\}} + b_T^\gamma e^{-\gamma\delta T} I_{\{\varepsilon > T-\tilde{\tau}\}}\right] = K_a^\gamma e^{-\gamma\delta\tilde{\tau}} \int_0^{T-\tilde{\tau}} e^{-\gamma\delta\varepsilon} \lambda e^{-\lambda\varepsilon} d\varepsilon + b_0^\gamma e^{\gamma(\mu_b - \delta)T} \int_{T-\tilde{\tau}}^\infty \lambda e^{-\lambda\varepsilon} d\varepsilon = K_a^\gamma e^{-\gamma\delta\tau} \frac{\lambda}{\lambda+\gamma\delta} \left(1 - e^{-(\lambda+\gamma\delta)(T-\tilde{\tau})}\right) + b_0^\gamma e^{\gamma(\mu_b-\delta)T} e^{-\lambda(T-\tilde{\tau})}$ and substituting $\tilde{\tau}$ yields the result in (2.13).

[14] Condition $\mu_a = \mu_b$ implies that a process b_t/a_t is a martingale and $E[b_t/a_t] = b_0/a_0$.

[15] The value of the boundary liquidity rate $\bar{\lambda} = 1.193 \cdot 10^{-4}$ translates into the average expected delay of $\bar{\varepsilon} = 8382.4$ days which is substantially larger than the maturity 5 days, therefore, any limit order submission is extremely likely to result in converting into a market order at maturity.

Table 2.2: Baseline parameters of the market with imperfect liquidity.

Parameter	Value
initial prices	$a_0 = £1000,\ b_0 = £995$
drift	$\mu_a = \mu_b = 10\%$ p.a.
volatility	$\sigma_a = \sigma_b = 20\%$ p.a.
discount factor	$\delta = 5\%$ p.a.
time horizon	$T = 5$ days
delay intensity	$\lambda = 8$ day^{-1}
price correlation	$\rho = 0.1$
risk aversion	$\varphi = 0.01$

numerical integration with recursive adaptive Simpson quadrature rule to obtain the solution to the optimisation problem for medium levels of liquidity. I then examine the impact of various model parameters on the optimal decision of a trader. In order to increase the efficiency of the numerical procedure, I set a precision grid equal to £0.25 and discretise the solution obtained from the continuous time model.

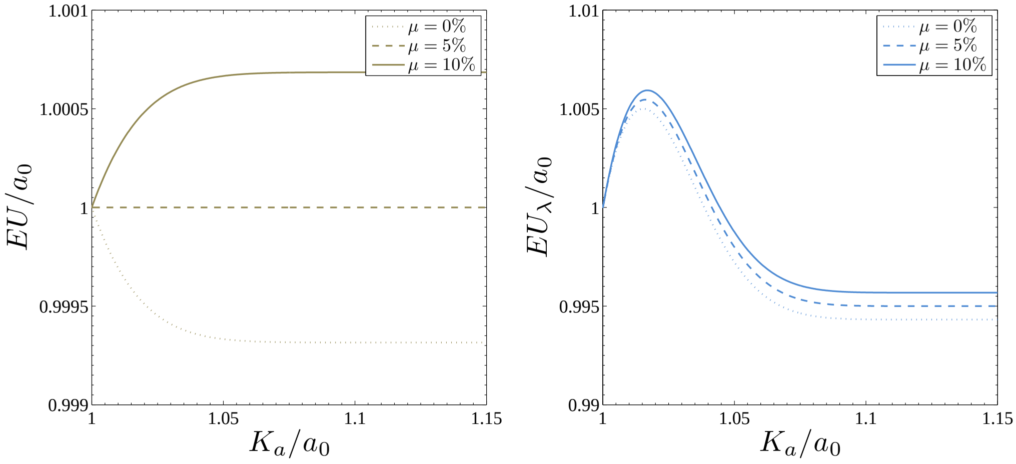
The optimal limit order price that corresponds to the baseline case is $K_a^* = £1008.75$. The average time-to-fill of this limit order is $\bar{\theta} = 44.8917$ days with the average first passage time of $\bar{\tau} = 44.7667$ days and $\bar{\varepsilon} = 0.125$ days mean delay in execution. The most likely delay is zero, however, the shape of exponential distribution impairs the relevance of mean value for my analysis, as was demonstrated in Section 2.4. The median time-to-fill is $\theta_{50\%} = 1.12$ days with a trivial median delay of $\varepsilon_{50\%} = 0.0866$ days. Lastly, the equilibrium spread, formally defined and discussed in Section 2.8, is equal to $\tilde{s} = £0.0175$ in the baseline case.[16]

In anticipation of the discussion about the effects of various market parameters on the optimal order placement I look at the shape of the expected quadratic utility function. A straightforward interpretation of the drift is to link it to the extent of traders' optimism or pessimism about future price movements based on past market performance. The greater the expected surge in the asset price, the higher the chances of a sell limit order receiving a fill before expiry.[17] Concerning trading in a risky asset, a drift higher than the discount factor δ implies a rise in stock price. This, in turn, deems limit orders more attractive for the investor who is willing to sell.

I compare the expected utilities of a risk-neutral and risk-averse traders. The expected profit that a risk-neutral agent attains by engaging with the limit order book is the same for any limit price

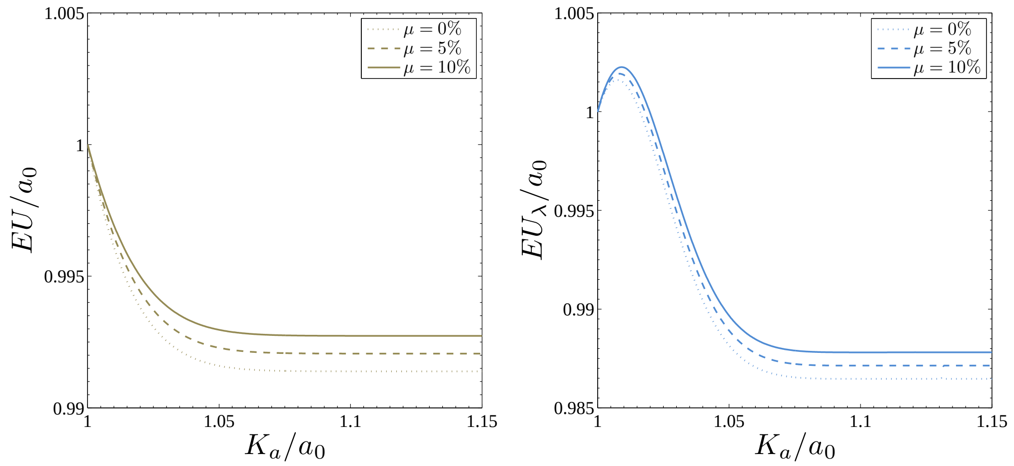
[16]The sensitivity of limit order strategy with respect to market parameters is studied for the size of the bid-ask spread at time zero $a_0 - b_0$ that exceeds the equilibrium level $\tilde{s}$. As Section 2.8 later notes, it is never optimal to use a market order at time zero under these circumstances, which eliminates the need to explicitly compare the optimal limit order expected utility and the payoff of an immediate market order.

[17]If $\mu \leq \sigma^2/2$ then the probability density function of the first passage time is defective and its integral over $[0, \infty)$ does not attain unity. Consequently, the unconditional probability that a limit order will never get filled is strictly positive.



(a) (b)

Figure 2.3: The expected utility of a sell limit order strategy for a risk-neutral trader as a function of the limit price: (a) no execution delay ($a_0 = b_0 = 1000$), (b) with a random execution delay ($a_0 = 1000$, $b_0 = 995$). Results are shown for parameters in Table 2.2.



(a) (b)

Figure 2.4: The expected utility of a sell limit order strategy for a risk-averse trader as a function of the limit price: (a) no execution delay ($a_0 = b_0 = 1000$), (b) with a random execution delay ($a_0 = 1000$, $b_0 = 995$). Results are shown for parameters in Table 2.2.

he might choose to submit, if the expected return of the underlying security equals the risk-free rate, as illustrated in Figure 2.3(a). If the price trend is downward sloping the best choice would be to sell at the current prevailing price, while an upward trend implies that a trader should use an infinitely high price. This relationship matches the result obtained by Iori et al. (2003), who prove for a market without spread that: (i) if $\mu > \delta$ the trader always waits until maturity ($K_a^* = \infty$), (ii) if $\mu < \delta$ the trader sells today with a market order, and (iii) if prices are martingales ($\mu = \delta$) the trader can submit any price $K_a \geq a_0$. In contrast, the expected utility of a trader selling via limit orders in a less liquid market retains a concavo-convex shape for a range of risk aversion coefficients, including $\varphi = 0$ (Figure 2.3(b) and Figure 2.4(b)).

2.7.2 Comparative Static Effects

Taking the effect of the drift parameter separately, notice that the optimal K_a^* in a situation without delay and no risk aversion is much more sensitive to the change in the price drift μ. In effect, the conclusion to be drawn from Figure 2.3(a) and Figure 2.4(a) is that perfect liquidity implies a binary choice: either to trade at the current quote or submit an infinitely high limit sell price. Once a random delay perturbs execution speed, the optimal limit price K_a^* increases linearly with μ, as Figure 2.5(a) clearly depicts, while perfect liquidity assumes a continuum of limit prices $K_a^* = \{a_0 \vee \infty\}$. Denote $\hat{\mu}$ the highest price drift for which the optimal selling strategy is a limit order at the current ask a_0. Other things being equal, the threshold drift, beyond which an agent switches to a more passive limit price is predictably moving rightward as risk aversion raises; this threshold actually lies in a negative domain for tolerant traders with $\varphi \leq 0.02$. Thereupon, in a market with imperfect liquidity a trader tends to choose a more passive strategy for certain values of the price drift, while the optimal decision of the same trader in an absolutely liquid market under equivalent circumstances is not uniquely defined.

One of the most crucial determinants of the optimal decision for a risk-averse trader is the bid-ask spread. The comparative static result in Figure 2.5(b) suggests that the limit price diminishes as the spread at the time of order placement increases. This reaction is induced by the penalty paid at maturity in the non-execution event: if the trader submits an order at a certain price K_a then the probability of its execution is fixed and the utility from using this limit order is declining whereby the spread increases. In their limit order market overview Parlour and Seppi (2008) argue that when spreads are tight, various types of agents use market orders, whereas only impatient traders resort to aggressive orders at times when spreads are wider. My stylised model emulates the tendency of incoming limit orders to cluster around best quotes when the spread is large, or, in the extreme, resort to a market order straightaway should the utility of the latter exceed the expected payoff from the optimal limit order strategy. In addition, Foucault et al. (2005) show through a dynamic equilibrium approach that impatient traders consume available liquidity, while more patient traders

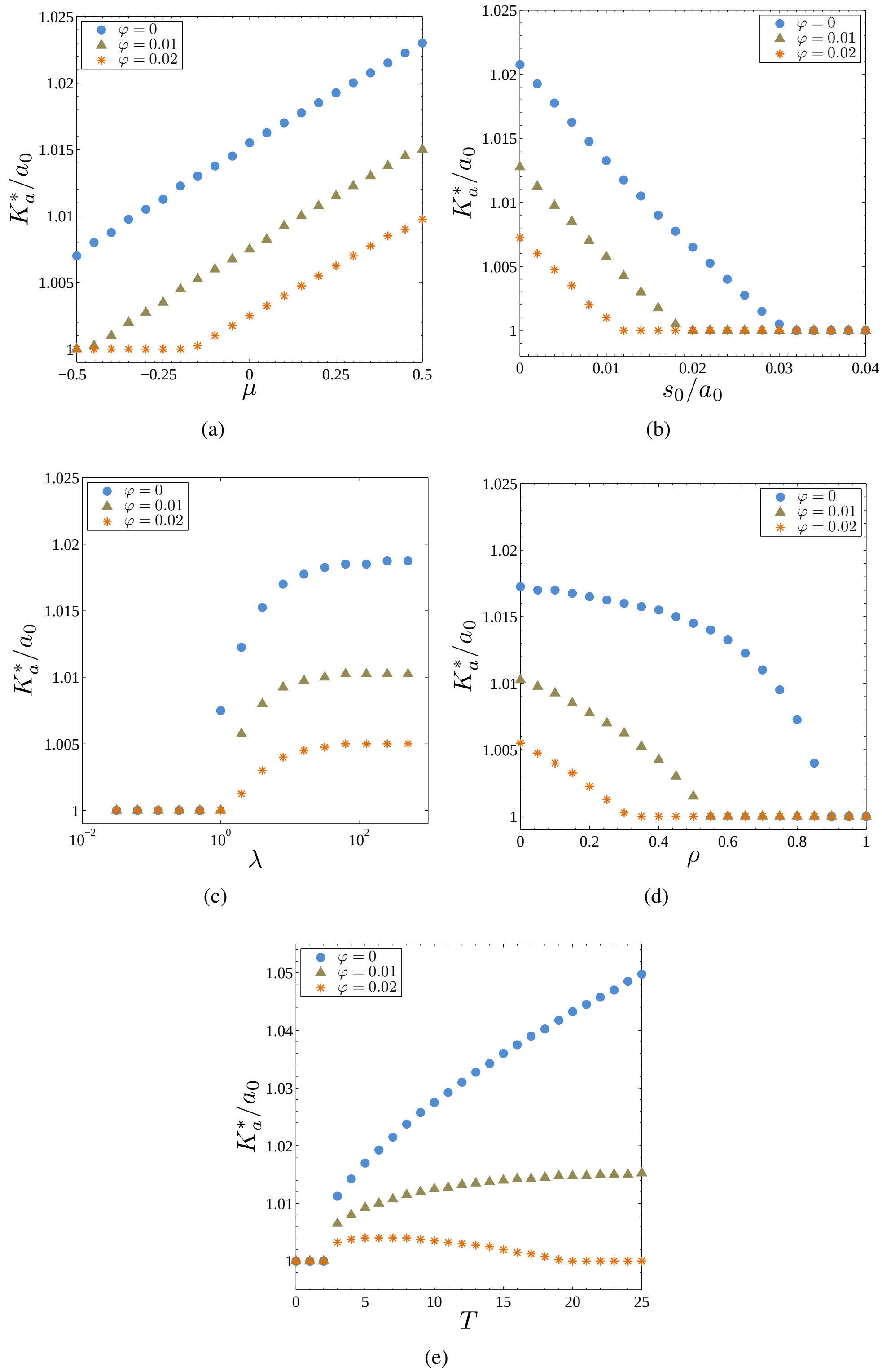


Figure 2.5: The optimal limit sell price for a risk-averse trader in a market with a random execution delay as a function of (a) the price drift, (b) the initial spread (in units of ask), (c) the execution delay intensity, (d) the bid-ask correlation, and (e) the time horizon. Results are shown for parameters in Table 2.2.

supply it. In my model, a relatively patient trader is characterised by a longer time horizon and higher risk tolerance. Therefore, a negative relationship between the optimal limit sell price and the spread size is anticipated for the trader who is bound by a time constraint.

Figure 2.5(c) demonstrates the effect of another driver of the optimal limit price – the execution delay intensity,[18] which characterises market liquidity. The more risk-averse the trader, the smaller the absolute impact of delay characteristic on the optimum. When liquidity is too low the limit price sticks at the best ask, while for intermediate levels of liquidity there is a positive relationship between λ and the optimal price K_a^*. This is an intuitively appealing result: an increase in the intensity rate λ augments the likelihood of a zero delay and reduces the variance of the time-to-fill, thus, other things being equal, leading to a shorter completion time of a trade. The inclusion of risk tolerance does not change the implication of imperfect liquidity, judging from a qualitative perspective: as anticipated, an increase in φ decreases the optimal K_a^*.

Further, I analyse the effect of the correlation between the paths of best bid and ask prices. In a market with a random delay higher correlation coefficient reduces gradually the optimal limit sell price that the agent submits (Figure 2.5(d)). If the bid and ask are moving in parallel and the limit order is not filled before maturity, then the agent's penalty is confined to the spread size. Essentially, the correlation controls the width of the spread. Equal drifts and a perfect correlation in bid and ask trajectories yields the situation of a constant spread. High correlation implies less uncertainty about the spread size in the future, so the trader gets impatient and prefers more aggressive strategies to avoid paying the spread at maturity since the time zero spread is large compared to, for instance, its equilibrium level. When the correlation is low, the spread can narrow or widen at maturity, therefore, the agent is more willing to trade with a limit order instead.

Order aggressiveness decreases as the maturity extends (Figure 2.5(e)) since longer time horizons imply higher chances that the ask a_t reaches the barrier represented by the trader's limit price K_a^*, in accordance with the behaviour shown in Figure 2.2. Bloomfield et al. (2005) report similar findings; in an experimental market they observe that liquidity-motivated traders rely primarily on limit orders but resolve to market orders as their trading horizons approach expiry. The length of a trading horizon is an attribute of the trader's impatience, hence my model implies that patient traders are inclined to supply liquidity to the market and place orders further away from the quotes. Moreover, the optimal strategy of a risk-averse trader in an illiquid market is apparently less respondent to a change in expiry time, so the impact of coefficient φ is that waiting for too long becomes equivalent to selling straightaway in terms of expected utility. This is also consistent with the special case of infinite time horizon discussed earlier in Section 2.6: when the time to maturity is very long, the order choice is more aggressive and not contingent upon maturity.

The findings illustrated in Figure 2.5 suggest that optimal limit order submissions in a market

[18]The exact formula for the power utility (2.8) for small $\lambda \leq \bar{\lambda}$ is used here to obtain the optimal submission price.

with imperfect liquidity tend to cluster reasonably close to the current best quote. This result is robust for a acceptable range of market parameters as confirmed by the sensitivity analysis presented above and relates to the humped shape of a limit order book extensively discussed in the literature (see Alfonsi et al. (2010); Biais et al. (1995); Potters and Bouchaud (2003); Rosu (2009); Weber and Rosenow (2005)).

2.8 The Equilibrium Spread

The width of the spread in financial markets is determined by the interactions of heterogeneous agents that populate them. In quote-driven markets the bid-ask spread is charged by the market maker in order to cover the expenses incurred by trading against better informed agents. The explanation of a positive spread in non-intermediated double auction markets is more subtle. The literature justifies it in several ways: information asymmetry which was briefly mentioned above, gravitational pull effect depicted by Cohen et al. (1981), variations in the state of the book and traders asset valuations, trading costs, which in turn comprise direct costs such as commissions, transfer taxes, order submission and account service fees, and indirect costs – the difference between the price at which the transaction was actually carried and a certain fair price. The random delay concept absorbs all these nuances which, in essence, generates the distinction between trading with a limit order and a market order.

Cohen et al. (1981) examine this issue in detail and show that a positive market spread is incumbent to this market microstructure. According to Cohen et al. (1981), the equilibrium spread in a dynamic trading system is "the bid-ask spread at which, for the next instant of time, the probability of the spread increasing is equal to the probability of the spread decreasing." They emphasise that equilibrium does not guarantee that the market will eventually settle at this equality, rather that it is more likely that the price will move towards this condition than in the opposite direction. This interpretation implicitly assumes that price movements already incorporate liquidity changes. On the other hand, Harris (2003, p. 304) advocates that in a non-intermediated market the equilibrium spread is "the spread which ensures that traders are indifferent between using a limit order and a market order." The relation between market parameters and the effect they have on traders' strategies jointly determine the bid-ask spread. Concrete market parameters motivate traders either to provide or to consume liquidity. I adopt the latter definition since it accommodates appropriately the notion of the execution delay that is central to my model. I further reveal that it is precisely the deviation from the perfect liquidity case inflicted by large delays in limit order execution that stipulates the spread size.

Definition 2.8.1 (Equilibrium Spread)**.** *In a dynamic trading process the equilibrium market spread is the bid-ask spread such that the expected utility from trading via a limit order at the optimal limit* *price is equal to the utility from an immediate market order. Let* $\tilde{s} = a_t - b_t, \forall$*, then the equilibrium spread* $\tilde{s}$ *satisfies*

$$\tilde{s} = a_0 - EU_\lambda(K_a^*; \vec{v}, \varphi \,\big|\, \tilde{s}), \tag{2.14}$$

where $EU_\lambda(K_a; \vec{v}, \varphi)$ *is a mean-variance utility function defined in (2.2) and* K_a^* *is the optimal limit price.*

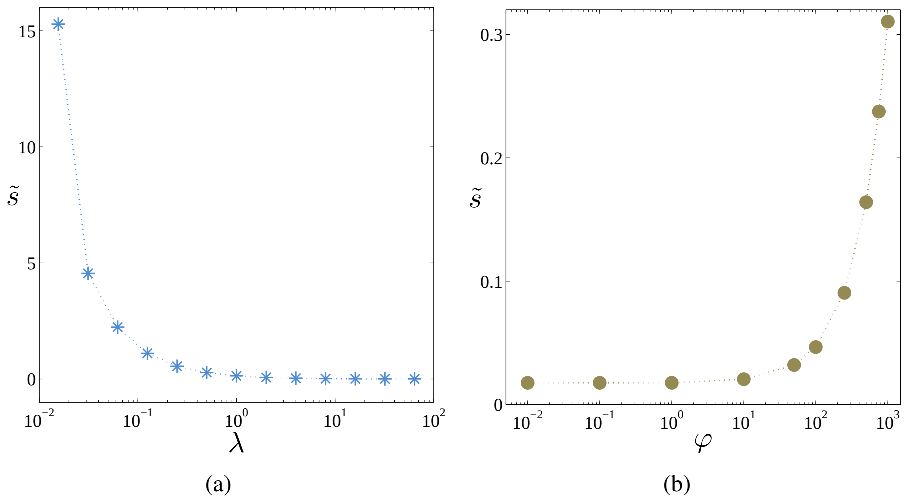


Figure 2.6: The equilibrium bid-ask spread as a function of (a) liquidity, and (b) risk-aversion. Results are shown for parameters in Table 2.2.

The equilibrium spread for the set of baseline parameters in Table 2.2 is computed via a numerical optimisation.[19] It has been discussed in the empirical literature that the main determinants of the spread size are competition for liquidity and risk aversion of market participants (Ranaldo, 2004). The higher is the competition among traders to provide liquidity, the tighter is the observed bid-ask spread, whereas the degree of traders' risk aversion stipulates a wider spread. As depicted in Figure 2.6(a), the equilibrium size of the spread $\tilde{s}$ diminishes and eventually attains zero as the level of liquidity λ increases, thereby decreasing the expected delay. Also, Figure 2.6(b) shows that the equilibrium spread increases, as expected, with the degree of risk aversion φ.

In my model, following the definition, when the actual bid-ask spread is higher than its equilibrium level, selling at the market brings very low profit, therefore, the trader will prefer to submit a limit order. At the same time, as Figure 2.5(b) depicts, the larger is the current spread, the more aggressive is the limit order strategy. This two-fold behaviour is grounded to the mechanism where

[19]The problem is solved in three steps. First, substitute $b_0 = a_0 - s$ and calculate the expected utility of limit orders submitted at various prices $K_a \geq a_0$ for a large range of spreads $s \geq 0$: $K_a^*(s) = \arg\max_{K_a \geq a_0} EU_\lambda(K_a; \vec{v}, \varphi \,\big|\, s)$. Second, determine the optimal limit price for each spread size and obtain an optimal limit price as a function of a spread $K_a^* = K_a^*(s)$ for a range $s \geq 0$. Third, find the pair of limit price and spread that satisfy condition (2.14).

the trader, that expects a price increase, avoids a market order at time zero, but places an aggressive limit order at $K_a = a_0$ which has the highest chance to trade soon, and uses a market order at maturity if the former does not get filled.

Although this result is obtained from the standpoint of a seller, in this setting the pattern for the buyer would be symmetrical. Moreover, since the limit orders are convertible in this model, a seller has to monitor both sides of the book. In other words, a market design where a limit order is executed immediately the moment it becomes the best price on its side requires a zero bid-ask spread.

2.9 Conclusions

In this chapter I develop a model of optimal order placement that is consistent with empirical findings of market microstructure literature. The central feature of this study is a random delay parameter, which defers limit order execution, characterises liquidity and justifies the non-transient bid-ask spread in double auction markets. With the aid of analytical and numerical solutions, I investigate the impact of execution delay on order submission patterns and the equilibrium bid-ask spread. The framework is based on observable market dynamics which facilitates potential application to actual data.

This framework simultaneously benefits from mathematical lightness and explains the trade-off between immediacy and a favourable transaction price. In contrast with standard first passage time models of trading, it captures the fundamental difference between the time required to reach the beginning of the queue on the relevant side of the market and the completion time of a trade. The distribution of the time-to-fill of limit orders conforms to the empirical distributions of trading times, and its variance decreases with liquidity. The mean-variance utility function permits adequate risk assessment for a strategy involving limit order trading, therefore, this approach allows to model the behaviour of heterogeneous investors. In addition, I demonstrate accounting for traders' risk aversion in strategy valuation eliminates submissions in the extremes of the book and thus provides empirically more coherent results. The results of comparative statics suggest that the introduction of a random delay factor alleviates the impact of various market conditions on the optimal limit price the trader submits and explains the clustering of limit order in the proximity of the current quotes. Notably, it is not only the magnitude but the mere presence of delay that alters the nature of the relationship.

Furthermore, I define the equilibrium spread as the bid-ask spread such that the expected utility from trading via a limit order at the optimal limit price is equal to the profit from an immediate market order. The model rationalises two more stylised facts: the equilibrium bid-ask spread decreases with liquidity, but increases with agents' risk aversion.

The solution is provided for a static problem but can be easily applied to multi-period submission steps and solved in the manner of Harris (1998). However, I expect the results not to change qualitatively once a trader is allowed to revise his strategy a finite number of times. Moreover, the model is tailored to moderate size trades, whereas large trades should be examined differently since they potentially entail serious price impacts when market orders are used to execute the trade. There is a separate branch of optimal trading literature on the order splitting issues which is adjacent to my framework (Alfonsi et al., 2010; Almgren and Chriss, 2000; Bertsimas and Lo, 1998; Løkka, 2013; Obizhaeva and Wang, 2013). Though most of order splitting is concerned with market order trading, in a recent paper by Guéant et al. (2012), the authors propose a novel approach of splitting a large trade using limit orders. Also, the choice of order type does not have to be restrictive. Cont and Kukanov (2012) discuss the properties of order splitting between market and limit orders and show that in case of a single exchange the amount that the trader executes via a limit order is independent of the total order size. These are potential directions for the future development of my framework.

Appendix A Proof of Proposition 2.4.1

First, I write a log-normal process as $a_t = a_0 e^{X_t}$ with $X_t = \left(\mu_a - \sigma_a^2/2\right)t + \sigma_a W_t$. Then the first passage time is $\tau = \inf\{t \geq 0 | X_t = x\}$ with $x = \ln(K_a/a_0)$. Denote the maximum of the X_t over a time period T as $M_T^X = \max\{X_t | 0 \leq t \leq T\}$. By the reflection principle $\forall\ x \geq 0$ the problem of optimal stopping time is equivalent to

$$P(\tau \leq t) = P\left(M_t^X \geq x\right).$$

The cumulative distribution of the maximum of a Brownian motion with drift ξ and volatility ν and $x \geq 0$ equals

$$P\left(M_t^X \leq x\right) = N\left(\frac{x - \xi t}{\nu\sqrt{t}}\right) - e^{2\xi x/\nu^2} N\left(\frac{-x - \xi t}{\nu\sqrt{t}}\right), \tag{2.15}$$

and the probability density is

$$f(t) = \frac{|x|}{\sqrt{2\pi\nu^2 t^3}} e^{\frac{-(x-\xi t)^2}{2\nu^2 t}}. \tag{2.16}$$

Since the prices follow a geometric Brownian motion, the first passage time has an inverse Gaussian distribution with the probability density function $f(\tau)$ given by (2.16) with parameters $\nu = \sigma_a$ and $\xi = \mu_a - \sigma_a^2/2$. The delay variable follows an exponential distribution with a positive parameter λ, so its probability density function is $f(\varepsilon) = \lambda e^{-\lambda\varepsilon}$. I denote the time-to-fill in the presence of a random delay in limit order execution by $\theta = \tau + \varepsilon$ and obtain its distribution:

$$\begin{aligned} P(\theta \leq t) &= \int_0^t f(\tau) \int_0^{t-\tau} f(\varepsilon) d\varepsilon d\tau \\ &= \int_0^t f(\tau) \int_0^{t-\tau} \lambda e^{-\lambda\varepsilon} d\varepsilon d\tau = \int_0^t f(\tau)\left(1 - e^{-\lambda(t-\tau)}\right) d\tau \\ &= \int_0^t f(\tau) d\tau - e^{-\lambda t} \int_0^t e^{\lambda\tau} f(\tau) d\tau. \end{aligned} \tag{2.17}$$

The first term of this expression is independent of λ and equals exactly the probability density function of the first passage time, while the second term accounts for the random delay. However, if parameter λ takes large values, then the integrand of the second term is not finite. I find the limit of the second term for high λ using Laplace's method.

Lemma 2.9.1. *Define an integral*

$$\mathcal{I}(\lambda) = \int_a^b h(\tau) e^{\lambda g(\tau)} d\tau, \tag{2.18}$$

where $[a, b]$ is a finite interval and functions $h(\tau)$ and $g(\tau)$ are continuous. Suppose function $g(\tau)$ attains a maximum on $[a, b]$ at either endpoint, $\tau_0 = a$ or $\tau_0 = b$, and is differentiable in a neigh-

borhood of τ_0, with $g'(\tau_0) \neq 0$ and $h(\tau_0) \neq 0$. Then the leading term of the asymptotic expansion of the integral (2.18) , as $\lambda \to +\infty$, is given by

$$\mathcal{I}(\lambda) = \frac{h(\tau_0) \cdot e^{\lambda g(\tau_0)}}{\lambda |g'(\tau_0)|}. \tag{2.19}$$

I apply Lemma 2.9.1 to functions $g(\tau) = \tau$ and $h(\tau) = \frac{|\alpha|}{\tau\sqrt{\tau}} n\left(\frac{\alpha+\beta\tau}{\sqrt{\tau}}\right)$ with integration limits $a = 0$, $b = t$. Function $g(\tau)$ attains the highest value at $\tau_0 = b = t$ and $g'(\tau_0) = 1$, therefore, I obtain from formula (2.19):

$$\int_0^t e^{\lambda\tau} \frac{|\alpha|}{\tau\sqrt{\tau}} n\left(\frac{\alpha+\beta\tau}{\sqrt{\tau}}\right) d\tau = \frac{\frac{|\alpha|}{t\sqrt{t}}\, n\left(\frac{\alpha+\beta t}{\sqrt{t}}\right) \cdot e^{\lambda t}}{\lambda \cdot |1|} = e^{\lambda t} \frac{|\alpha|}{\lambda t\sqrt{t}} \cdot n\left(\frac{\alpha+\beta t}{\sqrt{t}}\right). \tag{2.20}$$

With $\alpha = x/\sigma_a$ and $\beta = -A_1\sigma_a$, I now rewrite expression (2.17) as follows:

$$P(\theta \leq t) = P(\tau \leq t) - \frac{x/\lambda}{\sigma_a t\sqrt{t}}\, n\left(\frac{x - A_1\sigma_a^2 t}{\sigma_a\sqrt{t}}\right), \tag{2.21}$$

under on the probability space $(\mathcal{P}, \Omega, \mathcal{F})$ with $x = \ln\left(\frac{K_a}{a_0}\right)$ and $A_1 = \frac{\mu_a - \sigma_a^2/2}{\sigma_a^2}$.

Note that the probability density of first passage time $P(\tau \leq t)$ is defined in equation (2.16). Taking the derivative of the second term in expression (2.21) with respect to t, I arrive at the probability density function of limit order time-to-fill θ:

$$\begin{aligned} P(\theta \in dt) &= \frac{x}{\sigma_a t\sqrt{t}}\, n\left(\frac{x - A_1\sigma_a^2 t}{\sigma_a\sqrt{t}}\right)\left[1 - \frac{(x^2 - A_1^2\sigma_a^4 t^2 - 3t)/\lambda}{2\sigma_a^2 t^2}\right] \\ &= P(\tau \in dt)\left[1 - \frac{(x^2 - A_1^2\sigma_a^4 t^2 - 3t)/\lambda}{2\sigma_a^2 t^2}\right]. \end{aligned} \tag{2.22}$$

It follows immediately from (2.22) that when the expected delay approaches zero, time-to-fill is equivalent to the time it takes to reach the front of the queue: $\lim_{\lambda\to\infty} P(\theta \in dt) = P(\tau \in dt)$.

Appendix B Proof of Proposition 2.5.1

Deriving an analytical expression for the power function of the limit order strategy payoff requires to calculate $EG(K_a; \vec{v}, \gamma) \equiv E\left[K_a^\gamma e^{-\gamma\delta(\tau+\varepsilon)} I_{\{\tau+\varepsilon\leq T\}} + b_T^\gamma e^{-\gamma\delta T} I_{\{\tau+\varepsilon>T\}}\right]$. I split this expression into two components: $J_1 \equiv E\left[K_a^\gamma e^{-\gamma\delta(\tau+\varepsilon)} I_{\{\tau+\varepsilon\leq T\}}\right]$ and $J_2 \equiv E\left[b_T^\gamma e^{-\gamma\delta T} I_{\{\tau+\varepsilon>T\}}\right]$. Assuming that the stopping time and delays are independent, I implement the integrated expectations formula,[20] and

[20] $E(X) = E(E(X|Y))$

arrive at the following expression:

$$\begin{aligned} J_1 &= K_a^\gamma E\left[e^{-\gamma\delta(\tau+\varepsilon)} I_{\{\tau\le T\}} I_{\{\varepsilon\le T-\tau\}}\right] = K_a^\gamma E\left[e^{-\gamma\delta\tau} I_{\{\tau\le T\}} E\left[e^{-\gamma\delta\varepsilon} I_{\{\varepsilon\le T-\tau\}} \middle| \tau\right]\right] \\ &= K_a^\gamma \int_0^T e^{-\gamma\delta\tau} f(\tau) \int_0^{T-\tau} e^{-\gamma\delta\varepsilon} f(\varepsilon)\, d\varepsilon d\tau. \end{aligned} \tag{2.23}$$

The probability density function of the first passage time $f(\tau)$ is given in (2.16); the delay variable follows an exponential distribution with $f(\varepsilon) = \lambda e^{-\lambda\varepsilon}$. Therefore, I simplify the integral in (2.24):

$$\begin{aligned} \int_0^T e^{-\gamma\delta\tau} f(\tau) \int_0^{T-\tau} e^{-\gamma\delta\varepsilon} f(\varepsilon)\, d\varepsilon d\tau &= \int_0^T e^{-\gamma\delta\tau} f(\tau) \left[\int_0^{T-\tau} e^{-\gamma\delta\varepsilon} \lambda e^{-\lambda\varepsilon} d\varepsilon\right] d\tau \\ &= \int_0^T e^{-\gamma\delta\tau} f(\tau) \frac{\lambda}{\lambda+\gamma\delta}\left(1 - e^{-(\lambda+\gamma\delta)(T-\tau)}\right) d\tau \\ &= \frac{\lambda}{\lambda+\gamma\delta}\left[\int_0^T e^{-\gamma\delta\tau} f(\tau)\, d\tau - e^{-(\lambda+\gamma\delta)T} \int_0^T e^{\lambda\tau} f(\tau)\, d\tau\right]. \end{aligned}$$

Using equation (2.16) with $\nu = \sigma_a$ and $\xi = \mu_a - \sigma_a^2/2$, I rewrite the first term as

$$\begin{aligned} J_1 &= K_a^\gamma \frac{\lambda}{\lambda+\gamma\delta}\left[\int_0^T e^{-\gamma\delta\tau} \frac{|x|}{\sigma_a \tau\sqrt{\tau}} n\left(\frac{x - \left(\mu_a - \sigma_a^2/2\right)\tau}{\sigma_a\sqrt{\tau}}\right) d\tau\right. \\ &- \left. e^{-(\lambda+\gamma\delta)T} \int_0^T e^{\lambda\tau} \frac{|x|}{\sigma_a \tau\sqrt{\tau}} n\left(\frac{x - \left(\mu_a - \sigma_a^2/2\right)\tau}{\sigma_a\sqrt{\tau}}\right) d\tau\right]. \end{aligned} \tag{2.24}$$

Since the following equality holds for the normal density

$$e^{-\phi t} n\left(\frac{\alpha+\beta t}{\sqrt{t}}\right) = e^{-\alpha\beta+\alpha\sqrt{\beta^2+2\phi}} n\left(\frac{\alpha + t\sqrt{\beta^2+2\phi}}{\sqrt{t}}\right), \tag{2.25}$$

with $\alpha = x/\sigma_a, \beta = -(\mu_a - \sigma_a^2/2)/\sigma_a$ and $\phi = \gamma\delta$ I rearrange the first component of J_1 as

$$\int_0^T e^{-\gamma\delta\tau} \frac{|x|}{\sigma_a \tau\sqrt{\tau}} n\left(\frac{x - \left(\mu_a - \sigma_a^2/2\right)\tau}{\sigma_a\sqrt{\tau}}\right) d\tau = \frac{|x|}{\sigma_a} \int_0^T \frac{1}{\tau\sqrt{\tau}}\left[e^{x(A_1+A_2)} n\left(\frac{x + A_2\sigma_a^2\tau}{\sigma_a\sqrt{\tau}}\right)\right] d\tau,$$

where $A_1 = \frac{\mu_a - \sigma_a^2/2}{\sigma_a^2}$ and $A_2 = \frac{\sqrt{(\mu_a-\sigma_a^2/2)^2 + 2\gamma\delta\sigma_a^2}}{\sigma_a^2}$.

Finally, using the identity

$$\int_0^T \frac{1}{t\sqrt{t}} n\left(\frac{\alpha+\beta t}{\sqrt{t}}\right) dt = \frac{1}{|\alpha|}\left[N\left(\frac{-|\alpha|}{\sqrt{T}} - \operatorname{sgn}(\alpha)\beta\sqrt{T}\right) + e^{-2\alpha\beta} N\left(\frac{-|\alpha|}{\sqrt{T}} + \operatorname{sgn}(\alpha)\beta\sqrt{T}\right)\right], \tag{2.26}$$

substituting $\alpha = x/\sigma_a, \beta = A_2\sigma_a$ and $K_a \geq a_0$, I get

$$\frac{|x|}{\sigma_a}\int_0^T \frac{1}{\tau\sqrt{\tau}}\left[e^{x(A_1+A_2)}n\left(\frac{x + A_2\sigma_a^2\tau}{\sigma_a\sqrt{\tau}}\right)\right]d\tau = \left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x - A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x + A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right). \tag{2.27}$$

The second integral in J_1 is modified via the following identity which holds for the values $\lambda \leq \beta^2/2$:[21]

$$e^{\lambda t}n\left(\frac{\alpha+\beta t}{\sqrt{t}}\right) = e^{-\alpha\beta+\alpha\sqrt{\beta^2-2\lambda}}n\left(\frac{\alpha + t\sqrt{\beta^2 - 2\lambda}}{\sqrt{t}}\right). \tag{2.28}$$

Applying this equality to the second term in (2.24), I obtain

$$\int_0^T e^{\lambda\tau}\frac{|x|}{\sigma_a\tau\sqrt{\tau}}n\left(\frac{x - \left(\mu_a - \sigma_a^2/2\right)\tau}{\sigma_a\sqrt{\tau}}\right)d\tau = \frac{|x|}{\sigma_a}\int_0^T \frac{1}{\tau\sqrt{\tau}}\left[e^{x(A_1+A_3)}n\left(\frac{x + A_3\sigma_a^2\tau}{\sigma_a\sqrt{\tau}}\right)\right]d\tau,$$

where $A_3 = \frac{\sqrt{(\mu_a - \sigma_a^2/2)^2 - 2\lambda\sigma_a^2}}{\sigma_a^2}$. Using formula (2.26) with $\alpha = x/\sigma_a, \beta = A_3\sigma_a$ and $K_a \geq a_0$

$$\frac{|x|}{\sigma_a}\int_0^T \frac{1}{\tau\sqrt{\tau}}\left[e^{x(A_1+A_3)}n\left(\frac{x + A_3\sigma_a^2\tau}{\sigma_a\sqrt{\tau}}\right)\right]d\tau = \left(\frac{K_a}{a_0}\right)^{A_1+A_3} N\left(\frac{-x - A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_3} N\left(\frac{-x + A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right). \tag{2.29}$$

Combining results (2.28) and (2.30) I arrive at the final expression for the first term:

$$\begin{aligned} J_1 &= K_a^{\gamma}\frac{\lambda}{\lambda+\gamma\delta}\left[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x - A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x + A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right. \\ &- \left. e^{-(\lambda+\gamma\delta)T}\left(\left(\frac{K_a}{a_0}\right)^{A_1+A_3} N\left(\frac{-x - A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_3} N\left(\frac{-x + A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right)\right]. \end{aligned}$$

Note that the constraint $\bar{\lambda} \equiv \left(\mu_a - \sigma_a^2/2\right)^2/2\sigma_a^2$ coincides with one of the two natural parameters of an inverse Gaussian distribution. Given log-normality of asset prices, the first passage time of $\tau = \inf\{t \geq 0 : a_t = K_a\}$ has an inverse Gaussian distribution which belongs to the exponential family. The set of values for which the probability density function is finite on the entire support is called the natural parameter space. In particular, for the random variable with the inverse Gaussian probability density $f_X(x;\omega_1,\omega_2)$, there are two natural parameters $\eta = \{-\frac{\omega_2}{2\omega_1^2}; -\frac{\omega_2}{2}\}$.

Before calculating the second term J_2, I rewrite the bid and ask equations (2.3) and (2.4) to

[21] $e^{\lambda t}n\left(\frac{\alpha+\beta t}{\sqrt{t}}\right) = e^{\lambda t}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\alpha+\beta t}{\sqrt{t}}\right)^2} = \frac{1}{\sqrt{2\pi}}e^{-\frac{\alpha^2+\beta^2t^2+2\alpha\beta t-2\lambda t^2}{2t}} = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\alpha+\sqrt{\beta^2-2\lambda t}}{\sqrt{t}}\right)^2-\alpha\beta+\alpha\sqrt{\beta^2-2\lambda t}} = e^{-\alpha\beta+\alpha\sqrt{\beta^2-2\lambda}}n\left(\frac{\alpha+t\sqrt{\beta^2-2\lambda}}{\sqrt{t}}\right)$

express these processes in terms of a two-dimensional Brownian motion (W_1, W_2):

$$da_t = a_t\left(\mu_a dt + \bar{\sigma_a} d\bar{W}_t\right)$$
$$db_t = b_t\left(\mu_b dt + \bar{\sigma_b} d\bar{W}_t\right),$$

where $\bar{\sigma_a} = (\sigma_a,\ 0)$ and $\bar{\sigma_b} = \left(\sigma_b\rho,\ \sigma_b\sqrt{1-\rho^2}\right)$. Therefore,

$$a_t = a_0 e^{(\mu_a - |\bar{\sigma_a}|^2/2)t + \bar{\sigma_a}\bar{W}_t}$$
$$b_t = b_0 e^{(\mu_b - |\bar{\sigma_b}|^2/2)t + \bar{\sigma_b}\bar{W}_t}.$$

Notice that if b_t is a log-normal process then b_t^γ is also log-normally distributed. Applying Ito's formula to this process I aarive at

$$db_t^\gamma = b_t^\gamma\left(\gamma\mu_b + \frac{\gamma(\gamma-1)}{2}\sigma_b^2\right)dt + \gamma b_t^\gamma \bar{\sigma}_b d\bar{W}_t. \tag{2.30}$$

And applying Ito's lemma once again I get $b_T^\gamma = b_0^\gamma e^{(\gamma\mu_b - \gamma\sigma_b^2/2)T + \gamma\bar{\sigma}_b\bar{W}_T}$. Now I rewrite the second term

$$\begin{aligned}
J_2 &= e^{-\gamma\delta T} E\left[b_0^\gamma e^{(\gamma\mu_b + \gamma(\gamma-1)/2\sigma_b^2)T + \gamma\bar{\sigma}_b\bar{W}_T - \gamma^2\sigma_b^2/2T} \cdot I_{\{\varepsilon > T-\tau\}} \cdot (I_{\{\tau > T\}} + I_{\{\tau \le T\}})\right] \\
&= b_0^\gamma\ e^{\gamma(\mu_b + (\gamma-1)\sigma_b^2/2 - \delta)T}\ E^*\left[I_{\{\tau > T\}} + I_{\{\varepsilon > T-\tau\}} I_{\{\tau \le T\}}\right] \\
&= b_0^\gamma\ e^{\gamma(\mu_b + (\gamma-1)\sigma_b^2/2 - \delta)T}\ \left(E^*\left[I_{\{\tau > T\}}\right] + E^*\left[I_{\{\tau \le T\}} E^*\left[I_{\{\varepsilon > T-\tau\}}\middle|\tau\right]\right]\right) \\
&= b_0^\gamma\ e^{\gamma(\mu_b + (\gamma-1)\sigma_b^2/2 - \delta)T}\ \left(\int_T^\infty f(\tau)\,d\tau + \int_0^T f(\tau)\int_{T-\tau}^\infty f(\varepsilon)\,d\varepsilon d\tau\right),
\end{aligned} \tag{2.31}$$

where expectation is calculated under probability measure P^* defined by Radon-Nikodym derivative $\eta_T = \frac{dP^*}{dP} = e^{\gamma\bar{\sigma}_b W_T - \gamma^2|\bar{\sigma}_b|^2/2T}$. As Girsanov theorem states, $\bar{W}_t^* = \bar{W}_t - \gamma\bar{\sigma}_b t$ is a two-dimensional Brownian motion under P^*, so I rewrite the process X_t defined in Appendix A as

$$X_T = (\mu_a - \sigma_a^2/2)T + \bar{\sigma}_a\left(\bar{W}_T^* + \gamma\bar{\sigma}_b T\right) = (\mu_a - \sigma_a^2/2 + \gamma\rho\sigma - a\sigma_b)T + \bar{\sigma}_a\bar{W}_T^*,$$

implying a drift $\xi^* = \mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b$ under P^*.

The first integral in (2.31) is the probability that the first passage time exceeds horizon T, which is given in formula (2.15) for $\nu = \sigma_a$ and ξ^*:

$$\int_T^\infty \frac{|x|}{\sigma_a\tau\sqrt{\tau}} n\left(\frac{x - \left(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b\right)\tau}{\sigma_a\sqrt{\tau}}\right)d\tau = N\left(\frac{x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) - \left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right), \tag{2.32}$$

where $A_4 = \frac{\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b}{\sigma_a^2}$. Furthermore, simplifying the last term in (2.31) yields

$$\begin{aligned}\int_0^T f(\tau)\int_{T-\tau}^{\infty} f(\varepsilon)\,d\varepsilon d\tau &= \int_0^T f(\tau)\left[\int_{T-\tau}^{\infty}\lambda e^{-\lambda\varepsilon}d\varepsilon\right]d\tau \\ &= \int_0^T f(\tau)\,e^{-\lambda(T-\tau)}d\tau = e^{-\lambda T}\int_0^T e^{\lambda\tau} f(\tau)\,d\tau.\end{aligned}$$

I substitute the probability density function $f(\tau)$ with volatility σ_a and drift $\xi^* = \mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b$ and using property (2.28) obtain

$$\int_0^T e^{\lambda\tau}\frac{|x|}{\sigma_a\tau\sqrt{\tau}} n\left(\frac{x - \left(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b\right)\tau}{\sigma_a\sqrt{\tau}}\right)d\tau = \frac{|x|}{\sigma_a}\int_0^T \frac{1}{\tau\sqrt{\tau}}\left[e^{x(A_4+A_5)} n\left(\frac{x + A_5\sigma_a^2\tau}{\sigma_a\sqrt{\tau}}\right)\right]d\tau, \tag{2.33}$$

where $A_5 = \frac{\sqrt{(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b)^2 - 2\lambda\sigma_a^2}}{\sigma_a^2}$ and $\lambda \leq (\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b)^2/2\sigma_a^2$. Given the parameters values $\alpha = x/\sigma_a$, $\beta = A_5\sigma_a$ and $K_a \geq a_0$, I write an explicit expression for this integral as shown in formula (2.26)

$$\begin{aligned}\frac{|x|}{\sigma_a}\int_0^T \frac{1}{\tau\sqrt{\tau}}\left[e^{x(A_4+A_5)} n\left(\frac{x + A_5\sigma_a^2\tau}{\sigma_a\sqrt{\tau}}\right)\right]d\tau &= \left(\frac{K_a}{a_0}\right)^{A_4+A_5} N\left(\frac{-x - A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) \\ &+ \left(\frac{K_a}{a_0}\right)^{A_4-A_5} N\left(\frac{-x + A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right).\end{aligned} \tag{2.34}$$

Using (2.32) and (2.34),

$$\begin{aligned}J_2 &= b_0^{\gamma}\, e^{\gamma\left(\mu_b + (\gamma-1)\sigma_b^2/2 - \delta\right)T}\left[N\left(\frac{x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) - \left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right. \\ &+ \left. e^{-\lambda T}\left(\left(\frac{K_a}{a_0}\right)^{A_4+A_5} N\left(\frac{-x - A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_4-A_5} N\left(\frac{-x + A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right)\right].\end{aligned}$$

Thus,

$$\begin{aligned}EG(K_a;\vec{v},\gamma) &= K_a^{\gamma}\frac{\lambda}{\lambda+\gamma\delta}\left[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x - A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x + A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right. \\ &- \left. e^{-(\lambda+\gamma\delta)T}\left(\left(\frac{K_a}{a_0}\right)^{A_1+A_3} N\left(\frac{-x - A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_3} N\left(\frac{-x + A_3\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right)\right] \\ &+ b_0^{\gamma}\, e^{\gamma\left(\mu_b + (\gamma-1)\sigma_b^2/2 - \delta\right)T}\left[N\left(\frac{x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) - \left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right. \\ &+ \left. e^{-\lambda T}\left(\left(\frac{K_a}{a_0}\right)^{A_4+A_5} N\left(\frac{-x - A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_4-A_5} N\left(\frac{-x + A_5\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right)\right],\end{aligned}$$

with $A_1 = \frac{\mu_a - \sigma_a^2/2}{\sigma_a^2}$, $A_2 = \frac{\sqrt{(\mu_a - \sigma_a^2/2)^2 + 2\gamma\delta\sigma_a^2}}{\sigma_a^2}$, $A_3 = \frac{\sqrt{(\mu_a - \sigma_a^2/2)^2 - 2\lambda\sigma_a^2}}{\sigma_a^2}$, $A_4 = \frac{\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b}{\sigma_a^2}$,

$A_5 = \frac{\sqrt{(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b)^2 - 2\lambda\sigma_a^2}}{\sigma_a^2}$ and $\forall\ \lambda \in \left[0, \bar{\lambda}\right]$,
where $\bar{\lambda} = \min\left\{\frac{(\mu_a - \sigma_a^2/2)^2}{2\sigma_a^2}, \frac{(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b)^2}{2\sigma_a^2}\right\}$.

Appendix C Proof of Proposition 2.6.1

I find the limit of the expected utility using Laplace's integral approximation method. Using the result in (2.20), I approximate the second integral in (2.24) and the integral in (2.33) for the cases when $\lambda \geq \beta^2/2$ and identity (2.28) does not hold. It is easy to show that for high values of λ the power utility is approximately

$$\begin{aligned} EG(K_a;\vec{v},\gamma) \quad &\cong \quad K_a^\gamma \frac{\lambda}{\lambda+\gamma\delta}\left[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x - A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x + A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right. \\ &- \quad \left. e^{-\gamma\delta T}\frac{x/\lambda}{\sigma_a T\sqrt{T}}\, n\left(\frac{x - A_1\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right] \\ &+ \quad b_0^\gamma\, e^{\gamma\left(\mu_b + (\gamma-1)\sigma_b^2/2 - \delta\right)T}\left[N\left(\frac{x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) - \left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right. \\ &+ \quad \left.\frac{x/\lambda}{\sigma_a T\sqrt{T}}\, n\left(\frac{x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right], \end{aligned} \tag{2.35}$$

where $x = \ln\left(\frac{K_a}{a_0}\right)$, $A_1 = \frac{\mu_a - \sigma_a^2/2}{\sigma_a^2}$, $A_2 = \frac{\sqrt{(\mu_a - \sigma_a^2/2)^2 + 2\gamma\delta\sigma_a^2}}{\sigma_a^2}$, $A_4 = \frac{\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b}{\sigma_a^2}$. Further, noting that constants A_1, A_2 and A_4 are independent of λ, I obtain that in a perfectly liquid market

$$\begin{aligned} \lim_{\lambda\to\infty} EG(K_a;\vec{v},\gamma) \quad &= \quad K_a^\gamma\left[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x - A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x + A_2\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right] \\ &+ \quad b_0^\gamma\, e^{\gamma(\mu_b + (\gamma-1)\sigma_b^2/2 - \delta)T}\left[N\left(\frac{x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right) - \left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x - A_4\sigma_a^2 T}{\sigma_a\sqrt{T}}\right)\right]. \end{aligned}$$

Appendix D Spread Non-Negativity

Rewrite the bid and ask processes defined in (2.3) and (2.4) as bi-dimensional ρ-correlated geometric Brownian motions

$$\begin{aligned} a_t &= a_0 e^{\left(\mu_a - |\bar{\sigma}_a|^2/2\right)t + \bar{\sigma}_a \bar{W}_t} \\ b_t &= b_0 e^{\left(\mu_b - |\bar{\sigma}_b|^2/2\right)t + \bar{\sigma}_b \bar{W}_t} \end{aligned}$$

where $\bar{W}_t = (W_{1t}, W_{2t})$, $\bar{\sigma}_a = (\sigma_a, 0)$ and $\bar{\sigma}_b = \left(\sigma_b\rho, \sigma_b\sqrt{1-\rho^2}\right)$. The probability of a negative spread is given by $P(b_t/a_t > 1)$ for any $t \in [0, T]$, therefore, I obtain

$$\frac{b_t}{a_t} = \frac{b_0}{a_0}\, e^{\left(\mu_b - \mu_a - (|\bar{\sigma}_b|^2 - \sigma_a^2)/2\right)t + (\bar{\sigma}_b - \bar{\sigma}_a)\cdot\bar{W}_t} \tag{2.36}$$

with $(\bar{\sigma}_b - \bar{\sigma}_a) = \left(\rho\sigma_b - \sigma_a, \sigma_b\sqrt{1-\rho^2}\right)$. Denote the drift of this process by $\mu_z = \mu_b - \mu_a - (\sigma_b^2 - \sigma_a^2)/2$ and the volatility by $\sigma_z = \sqrt{\sigma_a^2 + \sigma_b^2 - 2\rho\sigma_a\sigma_b}$, then the log-normal process in (2.36) is $b_t/a_t = b_0/a_0 e^{Z_t}$ with $Z_t = \mu_z t + \sigma_z \tilde{W}_t$. Further, denote the maximum of this Brownian motion with drift as $M_T^Z = \max\{Z_t | 0 \leq t \leq T\}$. The probability of b_t exceeding a_t at any time t between 0 and T is equivalent to the probability that M_T^Z will be higher than $z = \ln(a_0/b_0)$, or

$$P\left(M_T^Z \geq z\right) = N\left(\frac{-z + \mu_z T}{\sigma_z\sqrt{T}}\right) + \left(\frac{a_0}{b_0}\right)^{2\mu_z/\sigma_z^2} N\left(\frac{-z - \mu_z T}{\sigma_z\sqrt{T}}\right). \tag{2.37}$$

Regarding the analysis presented in Section 2.7, the probability of b_t crossing a_t until T, i.e. while the trader's limit order stays in the book is approximately 0.7945 for the baseline model parameters (Table 2.2). However, since the trader cannot revise his limit order strategy before maturity, all that matters for him is that the probability of the bid price b_t crossing the limit price K_a is lower than the probability of a limit order at the same price, or:

$$P\left(\theta_a \leq T\right) > P\left(\tau_b \leq T\right), \tag{2.38}$$

where $\theta_a = \tau_a + \varepsilon$ is the time-to-fill of a limit order at K_a (with $\tau_a = \inf\{t \geq 0 | a_t = K_a\}$ and a random delay ε) and $\tau_b = \inf\{t \geq 0 | b_t = K_a\}$ is the time when the bid reaches K_a. Define $M_T^{X_a} = \max\{X_t^a \big| a_t = a_0 e^{X_t^a} \cup 0 \leq t \leq T\}$ and $M_T^{X_b} = \max\{X_t^b \big| b_t = b_0 e^{X_t^b} \cup 0 \leq t \leq T\}$. Substituting the first probability from (2.17) and applying the reflection principle to both probabilities I arrive at the following condition

$$P\left(M_T^{X_a} \geq \ln\left(K_a/a_0\right)\right) - e^{-\lambda T}\int_0^T e^{\lambda\tau_a} f(\tau_a) d\tau_a > P\left(M_T^{X_b} \geq \ln\left(K_a/b_0\right)\right). \tag{2.39}$$

This condition holds in the numerical example in Section 2.7 provided that random delay is plausible $\varepsilon \ll T$.

Appendix E Utility Boundaries Analysis

In order to assess the range of values that the expected utility (2.2) takes for intermediate levels of liquidity, I compute its upper and lower bounds of $EG(K_a; \vec{v}, \gamma) \equiv E\left[K_a^\gamma e^{-\gamma\delta(\tau+\varepsilon)} I_{\{\tau+\varepsilon\leq T\}} + b_T^\gamma e^{-\gamma\delta T} I_{\{\tau+\varepsilon>T\}}\right]$

for $\bar{\lambda} < \lambda \ll \infty$. The general form of the integral I need to bound is

$$\int_0^T e^{-\lambda(T-\tau)} f(\tau) d\tau, \tag{2.40}$$

where the density is $f(\tau) = \frac{|\alpha|}{\tau\sqrt{\tau}} n\left(\frac{\alpha+\beta\tau}{\sqrt{\tau}}\right)$ and λ takes large values. It is easy to see that

$$\forall \tau \in [0, T]: \quad e^{-\lambda(T+\tau)} \leq e^{-\lambda(T-\tau)} \leq 1,$$

and since $f(\tau)$ is a bounded function I have

$$\int_0^T e^{-\lambda(T+\tau)} f(\tau) d\tau \leq \int_0^T e^{-\lambda(T-\tau)} f(\tau) d\tau \leq \int_0^T f(\tau) d\tau.$$

For the upper bound I apply identity (2.26)

$$\begin{aligned} \int_0^T f(\tau) d\tau &= \int_0^T \frac{|\alpha|}{\tau\sqrt{\tau}} n\left(\frac{\alpha+\beta\tau}{\sqrt{\tau}}\right) d\tau \\ &= N\left(\frac{-\alpha+\beta T}{\sqrt{T}}\right) + e^{-2\alpha\beta} N\left(\frac{-\alpha-\beta T}{\sqrt{T}}\right). \end{aligned} \tag{2.41}$$

In order to calculate the lower bound I first use property (2.25)

$$e^{-\lambda(T+\tau)} f(\tau) = e^{-\lambda T} \frac{|\alpha|}{\tau\sqrt{\tau}} \left[e^{-\lambda\tau}\, n\left(\frac{\alpha+\beta\tau}{\sqrt{\tau}}\right)\right] = e^{-\lambda T} \frac{|\alpha|}{\tau\sqrt{\tau}} \left[e^{-\alpha\beta+\alpha\sqrt{\beta^2+2\lambda}} n\left(\frac{\alpha+\tau\sqrt{\beta^2+2\lambda}}{\sqrt{\tau}}\right)\right],$$

then property (2.26) to get

$$\begin{aligned} \int_0^T e^{-\lambda(T+\tau)} f(\tau) d\tau &= |\alpha|\, e^{\alpha(\psi-\beta)-\lambda T} \int_0^T \frac{1}{\tau\sqrt{\tau}}\, n\left(\frac{\alpha+\psi\tau}{\sqrt{\tau}}\right) d\tau \\ &= e^{-\lambda T}\left[e^{\alpha(\psi-\beta)} N\left(\frac{-\alpha+\psi T}{\sqrt{T}}\right) + e^{-\alpha(\psi+\beta)} N\left(\frac{-\alpha-\psi T}{\sqrt{T}}\right)\right], \end{aligned} \tag{2.42}$$

where $\psi = \sqrt{\beta^2 + 2\lambda}$.

The full expression for expected power utility is

$$\begin{aligned} EG(K_a; \vec{v}, \gamma) &= K_a^\gamma \frac{\lambda}{\lambda+\gamma\delta}\left[\int_0^T e^{-\gamma\delta\tau} \frac{|x|}{\sigma_a\tau\sqrt{\tau}} n\left(\frac{x-\left(\mu_a-\sigma_a^2/2\right)\tau}{\sigma_a\sqrt{\tau}}\right) d\tau\right. \\ &\quad - \left.\underbrace{e^{-(\lambda+\gamma\delta)T} \int_0^T e^{\lambda\tau} \frac{|x|}{\sigma_a\tau\sqrt{\tau}} n\left(\frac{x-\left(\mu_a-\sigma_a^2/2\right)\tau}{\sigma_a\sqrt{\tau}}\right) d\tau}_{I_2}\right] \end{aligned} \tag{2.43}$$

$$
\begin{aligned}
&+ \quad b_0^{\gamma} e^{\gamma(\mu_b+(\gamma-1)\sigma_b^2/2-\delta)T} \left[\int_T^{\infty} \frac{|x|}{\sigma_a \tau \sqrt{\tau}} n\left(\frac{x - \left(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b\right)\tau}{\sigma_a \sqrt{\tau}} \right) d\tau \right. \\
&+ \quad \left. \underbrace{e^{-\lambda T} \int_0^T e^{\lambda\tau} \frac{|x|}{\sigma_a \tau \sqrt{\tau}} n\left(\frac{x - \left(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b\right)\tau}{\sigma_a \sqrt{\tau}} \right) d\tau \right]}_{I_4}.
\end{aligned}
$$

As shown in the the proof of Proposition 2.5.1, the first and the third integrals have analytical expression for all values of parameters. I apply formula (2.42) to the second term I_2 in (2.43) with $\alpha = x/\sigma_a$ and $\beta = -A_1\sigma_a$ determine its upper bound

$$
-I_2 \leq -e^{-(\lambda+\gamma\delta)T} \left[\left(\frac{K_a}{a_0}\right)^{A_1+A_6} N\left(\frac{-x + A_6\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_6} N\left(\frac{-x - A_6\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) \right], \tag{2.44}
$$

where $A_6 = \frac{\sqrt{(\mu_a - \sigma_a^2/2)^2 + 2\lambda\sigma_a^2}}{\sigma_a^2}$, and formula (2.41) to determine its lower bound

$$
-I_2 \geq -e^{-\gamma\delta T} \left[N\left(\frac{-x - A_1\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) + \left(\frac{K_a}{a_0}\right)^{2A_1} N\left(\frac{-x + A_1\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) \right]. \tag{2.45}
$$

Similarly, I apply formula (2.41) to the fourth term I_4 in (2.43) with $\alpha = x/\sigma_a$ and $\beta = -A_4\sigma_a$ to determine its upper bound

$$
I_4 \leq N\left(\frac{-x - A_4\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) + \left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x + A_4\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) \tag{2.46}
$$

and formula (2.42) to find the lower bound

$$
I_4 \geq e^{-\lambda T} \left[\left(\frac{K_a}{a_0}\right)^{A_4+A_7} N\left(\frac{-x + A_7\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) + \left(\frac{K_a}{a_0}\right)^{A_4-A_7} N\left(\frac{-x - A_7\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) \right], \tag{2.47}
$$

where $A_7 = \frac{\sqrt{(\mu_a - \sigma_a^2/2 + \gamma\rho\sigma_a\sigma_b)^2 + 2\lambda\sigma_a^2}}{\sigma_a^2}$.

Finally, substituting (2.44) and (2.46) I obtain an *upper bound* of expected power utility:

$$
\begin{aligned}
EG^U(K_a; \vec{v}, \gamma) \quad &= \quad K_a^{\gamma} \frac{\lambda}{\lambda + \gamma\delta} \left[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x - A_2\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x + A_2\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) \right. \\
&- \quad \left. e^{-(\lambda+\gamma\delta)T} \left(\left(\frac{K_a}{a_0}\right)^{A_1+A_6} N\left(\frac{-x + A_6\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) + \left(\frac{K_a}{a_0}\right)^{A_1-A_6} N\left(\frac{-x - A_6\sigma_a^2 T}{\sigma_a \sqrt{T}} \right) \right) \right] \\
&+ \quad b_0^{\gamma} e^{\gamma(\mu_b+(\gamma-1)\sigma_b^2/2-\delta)T}.
\end{aligned} \tag{2.48}
$$

Using results (2.45) and (2.47) I obtain a *lower bound* of expected power utility:

$$
\begin{aligned}
EG^L(K_a;\vec{v},\gamma) \;=\;& K_a^{\gamma}\frac{\lambda}{\lambda+\gamma\delta}\Bigg[\left(\frac{K_a}{a_0}\right)^{A_1+A_2} N\left(\frac{-x-A_2\sigma_a^2T}{\sigma_a\sqrt{T}}\right)+\left(\frac{K_a}{a_0}\right)^{A_1-A_2} N\left(\frac{-x+A_2\sigma_a^2T}{\sigma_a\sqrt{T}}\right)\\
&- e^{-\gamma\delta T}\left(N\left(\frac{-x-A_1\sigma_a^2T}{\sigma_a\sqrt{T}}\right)+\left(\frac{K_a}{a_0}\right)^{2A_1} N\left(\frac{-x+A_1\sigma_a^2T}{\sigma_a\sqrt{T}}\right)\right)\Bigg]\\
&+ b_0^{\gamma}e^{\gamma(\mu_b+(\gamma-1)\sigma_b^2/2-\delta)T}\Bigg[N\left(\frac{x-A_4\sigma_a^2T}{\sigma_a\sqrt{T}}\right)-\left(\frac{K_a}{a_0}\right)^{2A_4} N\left(\frac{-x-A_4\sigma_a^2T}{\sigma_a\sqrt{T}}\right)\\
&+ e^{-\lambda T}\left(\left(\frac{K_a}{a_0}\right)^{A_4+A_7} N\left(\frac{-x+A_7\sigma_a^2T}{\sigma_a\sqrt{T}}\right)+\left(\frac{K_a}{a_0}\right)^{A_4-A_7} N\left(\frac{-x-A_7\sigma_a^2T}{\sigma_a\sqrt{T}}\right)\right)\Bigg]
\end{aligned}
$$

with $A_1 = \frac{\mu_a-\sigma_a^2/2}{\sigma_a^2}$, $A_2 = \frac{\sqrt{(\mu_a-\sigma_a^2/2)^2+2\gamma\delta\sigma_a^2}}{\sigma_a^2}$, $A_4 = \frac{\mu_a-\sigma_a^2/2+\gamma\rho\sigma_a\sigma_b}{\sigma_a^2}$, $A_6 = \frac{\sqrt{(\mu_a-\sigma_a^2/2)^2+2\lambda\sigma_a^2}}{\sigma_a^2}$ and $A_7 = \frac{\sqrt{(\mu_a-\sigma_a^2/2+\gamma\rho\sigma_a\sigma_b)^2+2\lambda\sigma_a^2}}{\sigma_a^2}$.

The values of the power function $EG(K_a;\vec{v},\gamma)$ fall, by construction, between boundary functions (2.48) and (2.49). However, the interval given by $EG^U(K_a;\vec{v},\gamma)$ and $EG^L(K_a;\vec{v},\gamma)$ appears to be rather wide, and these boundary functions do not produce an accurate solution to the problem (2.2). Figure 2.7 indicates that these boundary functions have different monotonicity regions in terms of limit prices compared to the expected utility curve.

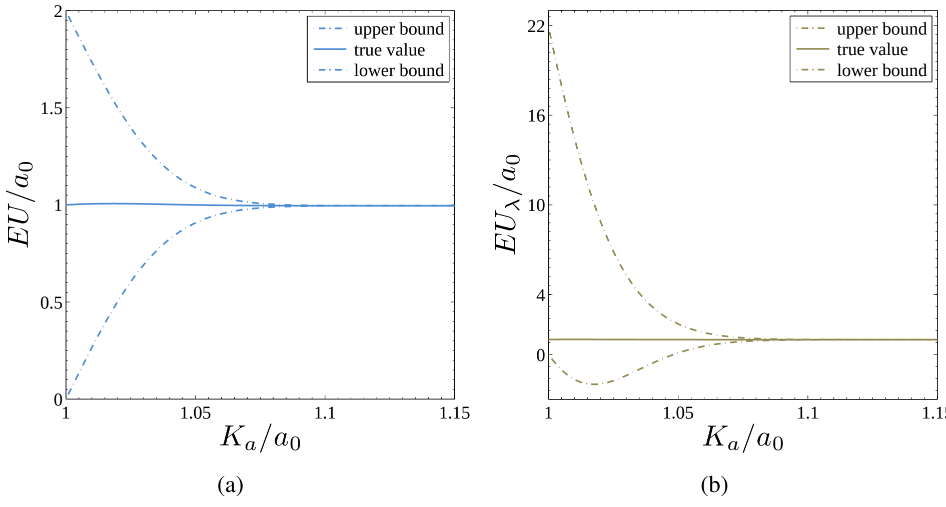


Figure 2.7: The expected utility of a sell limit order strategy for a risk-averse trader and with upper and lower bounds as functions of the limit price: (a) no execution delay ($a_0 = b_0 = 1000$), (b) with a random execution delay ($a_0 = 1000$, $b_0 = 995$). Results are shown for parameters in Table 2.2.

Chapter 3

The Luring Opacity of a Limit Order Market

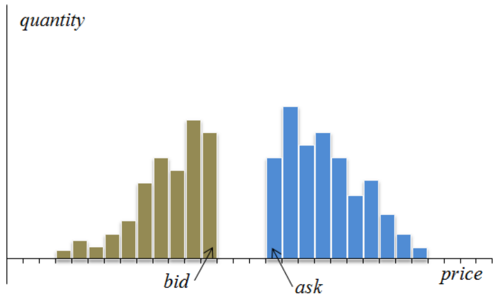
3.1 Introduction

Market transparency is the topic of ceaseless discussion in microstructure research and regulation as it perturbs market efficiency and the concept of fair pricing. There are two types of transparency: *pre-trade* transparency relates to publishing up-to-date information about quotes and orders, and *post-trade* transparency – to information about executed trades. Limit order markets, the majority of which operate through electronic trading books, are considered to be highly transparent trading vehicles. A typical electronic market is represented by a stream of market orders and two queues of limit orders: a queue of agents willing to sell an asset and a queue of agents willing to purchase it. Normally, most of the trading volume is concentrated around the best quotes – the bid (highest buying) and ask (lowest offer) prices, as depicted in Figure 3.1(a). A market order is executed immediately upon its arrival, while a limit order is available in the book until a matching order arrives or it is cancelled by the trader himself. Both market and limit order arrivals cause asset price movements, therefore, traders perceive new orders as information signals about the future asset value.

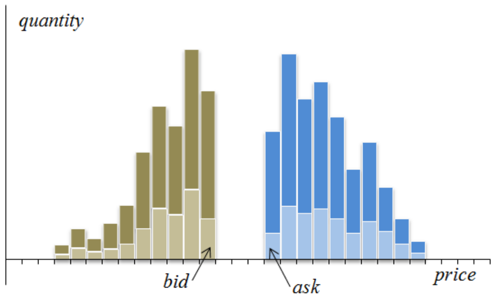
There are multiple ways in which traders can communicate their intentions to other market participants. In this respect, buying or selling large volumes of security requires comprehensive exposure management since it can produce a significant price impact[1] and affect execution prices. On the one hand, the major benefit of proactive trading via limit orders is in attracting traders with latent trading interests which, other things being equal, leads to faster trade execution and lower

[1] See studies by Cont et al. (2012); Hasbrouck (1991); Iori et al. (2003); Potters and Bouchaud (2003); Saar (2001) and Weber and Rosenow (2005) for comprehensive investigations about price impact in order-driven markets – statistical properties, shape and decomposition of the price impact function. Saar (2001), for instance, uncovers the asymmetry between buy and sell side impact and its determinants.

trading costs. On the other hand, substantive order exposure often entails costs which are caused by opportunistic or parasitic traders who build their strategies to profit at the expense of exposed traders. Parasitic traders are uninformed traders in the usual sense of the term since they do not posses any private information about the fundamental value of the asset.[2]



(a) The displayed limit order book.



(b) The actual limit order book.

Figure 3.1: The typical shape of an average limit order book.

Intuition suggests that agents with a good insight about true market conditions favour less transparent trading because their private knowledge is preserved better in such environments; while those agents who do not have information advantages usually benefit from transparency. Nevertheless, it is never possible to infer the full amount each market participant intends to buy or sell from available data even in a perfectly transparent exchange. Modern electronic double auction markets, with a few technical differences, augment this natural asymmetry by permitting traders to conceal partly or entirely their limit order size by submitting an *iceberg* order, which, as the name indicates, enables the trader to keep a fraction of his order size temporarily hidden.[3] Each iceberg order has a fixed limit price, a disclosed quantity – the total amount of shares to buy or sell, and a peak – the order size publicly displayed in the electronic trading book. Once the peak of an iceberg order is executed, the order is renewed by the same amount and the remaining quantity is reduced accordingly. In this manner iceberg orders assist exposure control and retain price priority with a sacrifice in time priority. Iceberg orders add depth to the displayed limit orders at given prices and, as a result, create two market layers: a visible volume and a shadow volume, where the latter is temporarily invisible and mimics the former (Figure 3.1(b)). Such microstructure, whereabout a part of the information about the current state of the book is not available to trading participants, belongs to the category of opaque markets.

[2]One type is *order anticipators* who aim to predict the impact of other traders' orders on the market price. They also classify into *front runners*, *sentiment-oriented technical traders* and *squeezers* depending on the sort of information they rely on. Another type is *bluffers*: *rumormongers* who disseminate false information and *price manipulators* who trade in order to confuse other traders.

[3]If the order size is completely omitted, the order is called hidden. This distinction, however, does not affect the narrative of this chapter and both terms are used somewhat interchangeably unless specified otherwise.

The depth of hidden layers in opaque markets has been repeatedly investigated from an empirical perspective by many researchers. On the basis of numerous studies covering different time spans on different trading venues the estimates of volumes hidden in limit order books range from reasonably low to high. The discrepancies in these estimates are partly due to specific microstructure characteristics of each market. Nonetheless, they largely suggest that hidden volume is not trivial, according to the findings collated in Table 3.1 and Figure 3.2.

Table 3.1: The summary of hidden volume estimates from empirical studies.

Study	Market	Year	Variable	Percent
D'Hondt et al. (2004)	Euronext	2002	depth at best 5	45%
Bessembinder et al. (2009)	Euronext Paris	2003	order volume	44%
Aitken et al. (2001)	ASX	1993	order volume	28%
Tuttle (2006)	Nasdaq	2002	inside depth	22%
Hasbrouck and Saar (2009)	INET	2004	all executed orders	15%
Frey and Sandås (2009)	XETRA	2004	non-marketable orders	9%
Esser and Mönch (2007)	XETRA	2002	total ask volume	8.24%

For example, Aitken et al. (2001) found that hidden orders represented approximately 28% of the volume on the Australian Stock Exchange in 1993. D'Hondt et al. (2004) document that hidden depth on Euronext Paris accounts for more than 45% of the total depth at the five best quotes and, according to Tuttle (2006), hidden liquidity in 100 Nasdaq stocks represented 22% of the inside depth in 2002. Drawing from more recent findings, hidden orders account for more than 15% of all orders executed on INET (Hasbrouck and Saar, 2009) and 9% of non-marketable orders in XETRA are iceberg orders (Frey and Sandås, 2009). On the extreme end of opacity spectrum lie *dark pools* with no market depth feed, where all trades are completely anonymous. Given lenient regulation of dark trading, these markets are steadily gaining popularity among large investors.

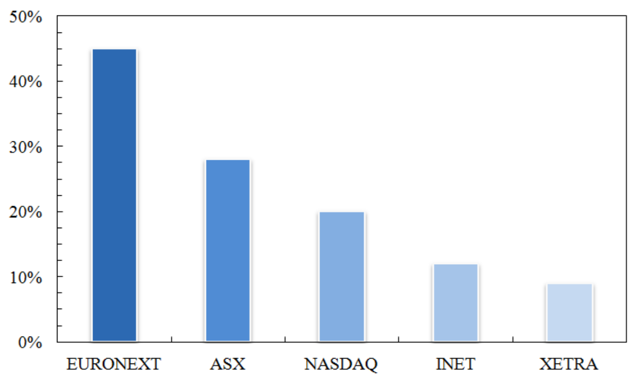


Figure 3.2: The empirical estimates of hidden liquidity in cumulative market volumes.

Although abundance of hidden liquidity is undeniable, there is little theoretical guidance on the role of and necessity for iceberg orders. Existing studies on hidden liquidity form two main branches: they either analyse the impact of iceberg orders on market quality, or devise optimal implementation schemes of iceberg orders while treating the hidden volume in the book as a priori beneficial. In this chapter I take a step aside from these two main directions of research and complement the literature by analysing the attractiveness of opaque markets to traders. Given that increasingly large volumes are traded on dark or semi-dark venues, it is necessary to comprehend how observable market conditions drive the willingness of traders to participate in such markets. Hence, in this chapter I develop a stylised model of trade execution in a market with hidden depth. I establish the optimal order submission timing of a risk-neutral trader in response to the observed order flow. Although the reduced transparency influences the trader's ability to predict market direction, he submits his order rather quickly, particularly, if the security is traded actively in the opaque market. At the same time, the trader seems to engage in riskier strategies in darker markets, being satisfied with a weaker belief to commit to trade. Furthermore, I demonstrate that the likelihood of choosing a wrong strategy due to market opacity escalates when transparency declines. This framework contributes to the existing literature by implementing a real option approach which to date remains novel in the market microstructure research area. In a broader sense this model relates to the topic of market fragmentation as it explains why many traders prefer to remain in lit markets, whereas others are lured by opacity.

The remainder of this chapter proceeds as follows. Section 3.2 reviews relevant theoretical and empirical literature and lays out the motivational grounds of the framework. Section 3.3 describes the design of a market in which a risk-neutral trader seeks to execute his trade. Section 3.4 provides the solution to the optimal stopping problem in the form of a threshold policy and carries out the sensitivity analysis with respect to various market conditions, followed by the error analysis in Section 3.5. Section 3.6 illustrates the main implications of the model with a numerical example, examining the sensitivity of the optimal strategy to the change in market conditions, order submission timing, and the probabilities of misjudging the demand for the security. Section 3.7 summarises the main findings achieved in this framework and outlines potential extensions.

3.2 Literature Review

In order to understand the consequences of imperfect market transparency induced by hidden liquidity, it is crucial to recognise its beneficiaries. Iceberg orders ascribe important flexibility to limit order usage since they enable agents to reduce market impact of their trades, which is particularly vital for large order sizes. However, iceberg orders assist better execution in terms of more favourable price, but at the expense of longer trading times. In essence, iceberg orders represent a special type of dynamic trading strategy. Large trade execution via iceberg orders is closely related to the order splitting problem when agents optimise the price and the fraction of the quantity to reveal at specific time intervals over their trading horizons.[4] In this context, any iceberg order is a naive splitting strategy since both the price and the quantity of each order renewal remain constant.

As informed traders usually strive to conceal their knowledge, it is convenient to think that these agents are predominant users of iceberg orders, treated as a specific type of limit orders. It is generally perceived that informed traders worry that their information advantage wears off as time goes by, hence they are expected to exhibit more aggressive behaviour in the market. For instance, Kaniel and Liu (2006) show by means of a modified Glosten-Milgrom (1985) type equilibrium model that, in contrast with the traditional assumption adopted in the literature, informed traders are prone to use limit orders subject to the time horizon allocated for execution. This is especially pronounced when the information they possess is long-lived. At the same time, iceberg orders are also more likely to be used by less informed traders since the latter have weaker predictions of future securities prices.

The empirical evidence on this aspect is blurred. Anand and Weaver (2004) establish that informed traders rely more frequently on hidden orders in case of low non-execution risks, given that they are usually very impatient and benefit most from managing price impact of their trades. On the contrary, Aitken et al. (2001) find that hidden orders are primarily used by uninformed traders to reduce adverse selection costs when picked off by parasite traders. In other words, the results from empirical research seem to deviate from market to market, suggesting no clear indication on the relationship between traders' information scope and their propensity to hide the true order size. The theoretical work by Madhavan et al. (2005) on this subject suggests that greater transparency can disincentivise traders and reduce liquidity, while Baruch (2005) shows how expanding public information in a limit order market with smart limit order traders and a dealer can enhance liquidity. In fact, the motivation to use iceberg orders is primarily driven by the liquidity demand rather than information, an argument supported by the work of DeWinne and D'Hondt (2007), Frey and Sandås (2009) and Pardo and Pascual (2012).

This conclusion, however, does not diminish the importance of information asymmetry and its effect on market performance. Higher market transparency mitigates to some extent information asymmetry among market participants, thus in principle lowering adverse selection costs incurred by traders and the volatility of prices by removing "noise". A number of empirical comparative studies investigate the repercussions of regulatory changes that reduce market transparency by facilitating the use of undisplayed orders. Anand and Weaver (2004) examine the data from the Toronto Stock Exchange, where hidden orders were banned in 1996 and then restored in 2002, and demonstrate that hidden liquidity does not adversely impact market quality as measured by quoted

[4]See, for instance, Almgren and Chriss (2000); Bernhardt and Hughson (1997); Løkka (2013); Obizhaeva and Wang (2013).

depth. Moreover, spreads and quoted depth are not affected by the abolition of hidden orders. In line with these results, Aitken et al. (2001) observe in the Australian Stock Exchange that setting a stricter pre-trade transparency rule, i.e. forcing to display larger fraction of the order size, thins the order book and slackens total trading volumes. The common insight that many studies seem to share is that tighter regulatory requirements motivate traders to consume liquidity rather than fill the limit order book, but not leave the market completely (DeWinne and D'Hondt, 2007; Frey and Sandås, 2009). Indeed, Frey and Sandås (2009) argue that although diluted market transparency requirements imply lower transaction costs and better execution, iceberg order users gain at the expense of limit order users. This tendency is perfectly plausible since markets with hidden liquidity are associated with longer time-to-fill of orders, according to Bessembinder et al. (2009). Accompanying the ample evidence on increased transparency events, studies that analyse the implications of reverse microstructure amendments, when markets go partially or completely dark, document that ensuing changes in market quality are of a smaller magnitude. In particular, Henderhott and Jones (2005) observe that spreads widen producing alongside a positive externality on parallel markets as order flow "spills over". Therefore, it is important to distinguish between cases of transparency weakening and reverse policies: the repercussions of these events may differ substantially, in the sense that liquidity restores and trading costs lower to a smaller extent once transparency is re-installed. Some markets are arguably more suitable than others for inferring the relationship between pre-trade transparency rules, traders' information sets and their propensity to hide due to disparities in market architectures. For instance, constraints on the maximum undisplayed fraction of order size determine indirectly the average order size and, if too high, they would discourage smaller traders from the market.

In the context of reduced transparency all traders are aware of hidden volumes sitting in the book and this certainly affects their strategic behaviour. The question is how accurately traders are able to evaluate the true market depth. Among possible indicators are aggressiveness of visible orders and the order arrival rate which, in turn, affects the competitiveness of the market. With respect to developed methodologies, Bessembinder et al. (2009) propose a logit and tobit approach to identify the presence of iceberg orders in the market by using publicly available variables such as order arrival rates, spread and recent transaction sizes, and visible market depth. In an extension of DeWinne and D'Hondt (2007) and Bessembinder et al. (2009), Frey and Sandås (2009) devise an efficient algorithm for iceberg order detection based on the examination of order flow patterns as well as iceberg peak sizes and the number of executed peaks: this algorithm misclassifies ordinary limit orders as iceberg orders with an average frequency of only 1%. Furthermore, recent literature documents a growing popularity of so-called fleeting orders that are withdrawn almost immediately after submission and are characterised by particularly large sizes. For instance, Hasbrouck and Saar (2009) find evidence in support for their *search hypothesis*: traders place aggressive orders inside the spread to catch hidden liquidity. Yet another way to discover the presence and the full size of an iceberg order is to use a large fill-or-kill market order: if sufficient volume is available the order will be executed rightaway and cancelled otherwise. It follows from here that if iceberg orders are detectable then choosing to trade via iceberg orders reveals more information than splitting via limit or market orders. Therefore, profusion of iceberg orders suggests that the *signaling property*, as Frey and Sandås (2009) refer to this phenomena, adds value to the strategy.

A number of theoretical frameworks model optimal exposure decisions of traders that operate in opaque markets, including, for instance, Esser and Mönch (2007) and Cebiroğlu and Horst (2011). Their results show that the probability of a certain trader to use a hidden order is higher, the smaller the revealed size and the larger the total size of his order. Among other determinants of order exposure distinguished in empirical research are the depth of a limit order book, the bid-ask spread, and the time of the day. Frey and Sandås (2009) document a feedback effect: the greater is the volume executed against iceberg orders the more market orders are used.

3.3 The Model

Consider a double auction market that operates through an electronic trading book, where limit orders are recorded but full volume is not known to the marketplace. The level of undisclosed liquidity generated by iceberg orders is never publicly revealed, not even post-execution. All traders participating in this market are aware of hidden liquidity but cannot measure it directly.

The shape of the limit order book and iceberg order processing are illustrated in Figure 3.3.[5] Let, for instance, seller X have an iceberg order recorded in the book with the displayed order size X_{disp}, the peak of the iceberg, and the remaining volume to trade, X_{hid}, hidden. This order is currently at the best ask, as depicted in Figure 3.3(a). All the orders on top of X have time priority since they were submitted earlier. Once these orders are executed, X_{disp} becomes the first in the queue (Figure 3.3(b)). When the visible order of trader X is fully filled (Figure 3.3(c)), this order is immediately renewed with the same amount X_{disp} and is put behind all visible orders at this price in the book; the hidden volume is then reduced accordingly $X'_{hid} = X_{hid} - X_{disp}$, as shown in Figure 3.3(d).

Consider a risk-neutral trader who intends to sell his position in this security market. Denote an exogenous latent variable γ that indicates the true demand for the asset in the opaque market and ultimately determines at what price the agent executes his trade. The trader intends to submit a sell limit order to this market at his preferred price, but if the demand is poor he switches to a market order instead.[6] Suppose that the timescale of the trader is narrow, so that the best bid B and

[5] In fact, my stylised framework abstracts from a concrete shape of a limit order book and models only the order flow. However, it is instructive for the analysis to examine the iceberg order mechanism in relation to the visible order flow.

[6] In real markets in this instance the trader would not only suffer from selling at a lower price, but usually he would

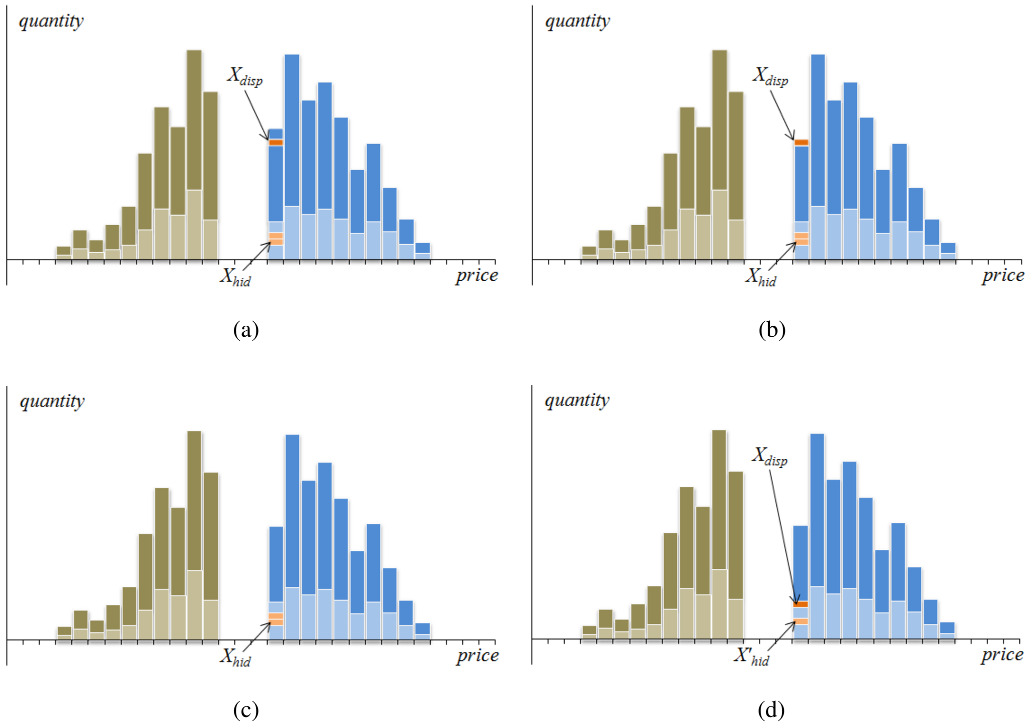


Figure 3.3: The mechanism of iceberg order peak release and gradual execution. Light colors indicate hidden volume.

the spread s remain constant, and the best ask is $A = B + s$.[7] If the demand for the asset is strong ($\gamma = 1$), the trader will sell the asset at a higher price A; if, on the contrary, the demand is weak ($\gamma = 0$), he will execute the trade at a lower price B. The order is submitted at a sunk cost $C \geq 0$ and cannot be revised or cancelled. These sunk costs represent a certain brokerage fee or, alternatively, one could think of this trader as a speculator who is conceiving a round-trip trade and C stands for his initial costs of purchasing the asset. The trader enters the market and monitors the order flow to decide when to place his order. I use the framework originally introduced by Thijssen et al. (2004) to model the investment choice under uncertainty in the context of information streams.

The order flow in this opaque market is composed of three order types: market orders, new passive orders, and renewals of old passive orders, i.e. of previously submitted iceberg orders. An additional buy order that appears in the limit order book indicates a good signal in terms of the true market demand, whereas an incoming sell order intensifies competition among sellers and hence is perceived as a bad signal. This interpretation applies to all market and limit orders. Orders arrive

incur additional switching costs such as limit order cancellation and other fees.

[7]Suppose that the market absorbs any volume at the best quotes, which can be achieved by a market maker who is responsible for guaranteeing sustainable depth at the quotes, or by assuming a substantial population of agents queuing at these quotes, as in the model of Seppi (1997).

at random points in time and follow a Poisson process with the rate λ. However, the market is not transparent, so the appearance of a passive order can be due to either a new limit order placement, with or without a hidden part, or a release of the subsequent iceberg order tranche. In the first case the total volume on the respective side of the market increases, whereas in the latter case the overall market depth remains unaltered. The probability of a change in the real market depth depends on the true state of the market γ. Assign $\theta \in (0, 1)$ to the probability of a new order submission, thereby $1 - \theta$ is the probability that an additional volume of the existing iceberg order is disclosed. This parameter is the clarity of the signal generated by the order flow and, in essence, it characterises the degree of market transparency: if the probability of new order submission is closer to unity, the hidden layer is insignificant, whereas with lower values of the probability θ hidden depth in this opaque market is substantial.

According to the theoretical and empirical findings of Handa et al. (2003), agents assess the thickness of the book by comparing current market depth on the buy side relative to the current market depth on the sell side, and both sides of the book are important in shaping the trading strategy of an agent. More precisely, as suggested by Ranaldo (2004), order aggressiveness depends positively on the volume available on the same side of the book and negatively – on the opposite side. Define the visible order imbalance h_t as the number of buy limit orders less sell limit orders displayed in the book at time t. Using the imbalance as a proxy, the trader evaluates the demand for security and hence the likelihood of selling via a passive strategy at a preferred price. If buy orders prevail ($h_t \gg 0$), then the trader believes that the demand exceeds supply and he can sell his asset at A. Conversely, if sell orders dominate the book ($h_t \ll 0$), the trader infers that the demand for this security is insufficient and he will execute at B to recover part of his costs. If the true demand for the asset in the opaque market is high, three events are possible at the next time instant:

(i) No order arrives with probability $1 - \lambda dt$.

(ii) A new buy order arrives with probability $\theta\lambda dt$. This directly indicates an increase in the demand, since market orders deplete the sell side and alleviate competition among asset suppliers, while fresh limit or iceberg orders extend the queue on the buy side and intensify competition among buyers, encouraging them to seek immediacy.

(iii) An old sell iceberg renews with probability $(1 - \theta)\lambda dt$, triggered by a buy market order that was placed at the previous time step. This indicates a decrease in the demand, since the trader cannot distinguish between old and new passive orders.

Therefore, the visible order imbalance h_t in a market with high demand is represented by the process:

$$dh_t = \begin{cases} 1 & \text{with probability} \quad \theta\lambda dt \\ 0 & \text{with probability} \quad 1-\lambda dt \\ -1 & \text{with probability} \quad (1-\theta)\lambda dt \end{cases} \tag{3.1}$$

Otherwise, if the true demand is low, the visible order flow mirrors this dynamics. The conditional expected increment in visible order imbalance is given by $E[dh_t|\gamma] = \text{sgn}\{\gamma - 0.5\} \cdot (2\theta - 1)$. Therefore, when the probability of a new order is 50%, the order arrival is not informative to the trader.

Due to uncertainty and lack of transparency, the trader cannot evaluate the true demand from market data, or predict future prices for that matter. He infers the demand for the asset from order arrivals that affect the visible buy-sell imbalance and constructs his belief p_t about the probability of a high demand for the asset. Upon entering the market the trader contemplates a prior belief p_0 that the demand for the asset is strong, which, in turn, implies a better chance of selling the security at a high price A. Over time, he learns what type of orders are predominantly submitted – buy or sell orders – and updates the posterior belief accordingly. Observing the order flow and gaining experience from the market, he adjusts his belief in more favourable execution according to Bayes rule:

$$p_t := p(h_t) = \frac{P(h_t|\gamma=1)P(\gamma=1)}{P(h_t|\gamma=1)P(\gamma=1) + P(h_t|\gamma=0)P(\gamma=0)} = \frac{\theta^{h_t} p_0}{\theta^{h_t} p_0 + (1-\theta)^{h_t}(1-p_0)},$$

or in a concise form

$$p_t = \frac{\theta^{h_t}}{\theta^{h_t} + \xi(1-\theta)^{h_t}}, \tag{3.2}$$

where $\xi = (1-p_0)/p_0$. Given that the conditional belief p_t is a monotonic function of order imbalance h_t for any fixed parameters p_0 and θ, there exists an inverse function:

$$h_t := h(p_t) = \frac{\ln\left(\frac{1-p_t}{p_t}\right) - \ln\xi}{\ln\left(\frac{1-\theta}{\theta}\right)}. \tag{3.3}$$

This implies that the problem can be formulated both in terms of visible order imbalance and in terms of the trader's posterior belief in a high demand and preferable execution.

In this framework, uncertainty is resolved over time, so eventually the trader will sell the asset. As soon as the trader places his order, the true state of the market γ is discovered: in case the demand is high, the limit order receives a fill; otherwise, if the demand is weak, the trader urges to sell at the best available market price. In reality it might take some time before the market reaches the limit price level $A = B + s$ or to execute the full amount of the limit order. However, I abstract from this to preserve the tractability of the model. The expected utility of the trader, conditional upon his ex ante belief in a high demand p_0, is expressed as follows:

$$U(h_t) = p(h_t) \cdot (B + s) + (1 - p(h_t)) \cdot B - C = p(h_t) \cdot s + B - C. \tag{3.4}$$

The trader's objective is to maximise his expected discounted utility. Taking into account a risk-free rate $r > 0$ and the available information set $\mathcal{F}_t$, the trader solves the following optimal stopping problem to determine the best time for the order placement:[8]

$$U^*(h_t) = \sup_{\tau \geq t} E\left[e^{-r\tau} U(h_\tau) \middle| \mathcal{F}_t \right]. \tag{3.5}$$

The structure of the model incorporates the dual nature of signals. The first aspect is the parameter λ which governs order arrival. This parameter is a measure for the *quantity* of orders, since $1/\lambda$ denotes the average time between two submissions. The other component is the probability θ of signal accuracy. This parameter is a measure for the *quality* of information embedded in the order flow. Given that the trader is learning about market conditions according to Bayes rule, the parameter θ explicitly affects the direction and the speed of long-run convergence of his belief. In a more transparent environment $\theta > 0.5$, the belief in a high demand for asset converges to one or to zero if the true market demand is strong or weak, respectively. Whereas in a darker market, $\theta < 0.5$, the belief in a high demand for asset reaches zero if the true market demand is strong and one if the true market demand is weak. If the signals are noisy, $\theta = 0.5$, the trader is better off by making the decision immediately by using his ex ante belief p_0. As will be demonstrated in Section 3.4, quantity and quality together determine the threshold belief in a high demand that the trader needs to reach to make the order placement optimal.

3.4 Optimal Policy

In this section I first derive optimality conditions and then discuss the impact of various factors on the trader's optimal submission time choice.

Proposition 3.4.1. *The utility maximisation problem of a risk-neutral trader (3.5) admits an analytical solution in the form of a threshold policy: the trader enters the market and submits his order as soon as his posterior belief in a strong demand for the asset exceeds the probability threshold*

$$p^* = \left[\left(\frac{s}{C - B} - 1 \right) \Pi + 1 \right]^{-1}, \tag{3.6}$$

[8]If there is no order imbalance $h_t = 0$, the trader's belief in a strong demand equals exactly his prior $p(0) = p_0$. Moreover, in the absence of learning, that is $p_t = \text{const}$, the optimal stopping problem essentially disappears, and the trader always submits his order immediately to resolve the uncertainty regarding true demand.

where

$$\Pi = \frac{r + (1-2\theta)\left(\lambda(1-2\theta) + \sqrt{r^2 + 2r\lambda + \lambda^2(1-2\theta)^2}\right)}{2r(1-\theta)}. \tag{3.7}$$

Proof. See Appendix A. □

Note that optimal belief threshold p^* can take any real value, but order imbalance is an integer. Thus the trader submits his order as soon as the net number of good signals reaches $h^* = \lceil h(p^*) \rceil$.[9]

I interpret the threshold policy as follows: while the trader's belief in a high demand p_t is below the threshold level p^*, he is unsure that the market will drift towards his sell limit price and is better off by refraining from placing orders. Once the visible order imbalance crosses the threshold h^*, the trader becomes convinced enough about favourable market conditions, pays the fee C and issues a sell limit order at A with a conversion to a market order at B in the worst case. Therefore, an increase in the threshold p^* signifies that the trader waits longer and gathers more signals before placing his order. I present below a comparative static analysis of the threshold p^* with respect to key market parameters embraced by this model. The analysis reveals that the threshold p^* is lower when buying orders prevail, the fees are higher, execution prices are lower, the spread is tighter, and transparency is higher. However, the impact of discount factor and liquidity level is determined in conjunction with the transparency level. The proof of the following propositions is obtained through simple calculus and is provided in Appendix B.

Proposition 3.4.2. *The threshold belief in a high demand for the asset, p^*, decreases with the selling price B and the spread s, and increases with the market entrance fee C.*

This result is in line with the objective of a risk-neutral agent to maximise the expected utility. If the market offers a better valuation of the asset in terms of prices A and B, then independently of the volume and imbalance, the agent will submit an order earlier, i.e. require a lower p^*. The size of the bid-ask spread determines the relative attractiveness of a limit order trade compared to execution at the market. If the spread is wide, the trader will act quicker in terms of his conviction to seize a potentially profitable opportunity. At the same time, if participating in the opaque market is costly, the trader will require stronger confidence in a sufficient demand, so he will wait longer until buy orders dominate substantially the sell side of the book.

Proposition 3.4.3. *The threshold belief in a high demand for the asset, p^*, increases with market liquidity λ in a more transparent market $\theta > 0.5$ and decreases in a darker market $\theta < 0.5$.*

If orders are submitted to the market more frequently but they are not indicative of the total market depth ($\theta < 0.5$), the trader will place his order sooner. Essentially, under these circumstances he will be indifferent what type of orders they are – market orders, ordinary limit orders or iceberg

[9] For any $z \in \mathbb{R}$, the ceiling function $f(z) = \lceil z \rceil$ is the smallest integer greater or equal to z.

orders. In other words, even if the demand is not going to increase, the fact that the security is actively traded can reassure the agent to place his order and expect an execution at a preferred price A. On the other hand, if the market is relatively transparent ($\theta > 0.5$) and the order flow is intensive, he waits longer until the book becomes more asymmetric.

Proposition 3.4.4. *The threshold belief in a high demand for the asset, p^*, decreases with the discount rate r in a more transparent market $\theta > 0.5$ and increases in a darker market $\theta < 0.5$.*

The discount rate impacts the optimal threshold p^* in the opposite direction as compared to liquidity: if transparency is low, then the larger the discount rate, the higher the threshold belief p^* and the starker visible imbalance towards the buy side is required by the trader to justify his order submission. When the trader eventually believes that the demand is sturdy and pays C to prompt his order into the book, he bears the risk of executing at $B \leq C$. If r is high, the loss $C - B$ in the adverse scenario declines as time goes by, hence the threshold increases. Similarly, given that this risk declines with opacity and guessing the real market depth makes the preferable execution more likely ceteris paribus, a higher discount rate makes the trader more impatient to collect his revenues.

Proposition 3.4.5. *The threshold belief in a high demand for the asset, p^*, increases with the degree of market transparency θ.*

On the one hand, if hidden depth is slim and the trader is initially pessimistic that a market can absorb his order. On the other hand, with higher degree of market transparency θ, the signal becomes more clear and for a given intensity of order arrival the trader will have to wait less to reach the threshold h^* in terms of actual time. This result appears counterintuitive but is explained by the double source of uncertainty in this model. Basic intuition suggests that in a market with a high demand when the signals embedded in the order flow become more informative, the order to trade will be placed sooner. However, in this model uncertainty has three dimensions: the degree of market transparency θ, the random arrival of orders λ, and the trader's uncertainty about the true state of the limit order book as a latent variable. While higher transparency reduces uncertainty about the depth of the book, it will never be eliminated completely.

The overall conclusion is that the optimal strategy implies that traders are attracted by markets with reduced transparency when the entry barriers are low, the spread is wide, execution prices and liquidity are high and opacity is significant. I further discuss these results in application to my numerical example in Section 3.6 and relate them to the evidence obtained from empirical literature.

3.5 Downsides of Market Opacity

The aim of this section is to offer several metrics that weigh up trader's chances of choosing an inappropriate strategy in a market with hidden liquidity. It is worth noting that the probabilities of such errors are often related to time constraints imposed on traders.[10]

So far I assumed that the trader is prepared to wait indefinitely until his belief in a high demand reaches the desired level. Suppose now the trader has a certain deadline by which he must decide whether to submit an order to the market with reduced transparency or not. Pressured by a deadline, the trader might avoid placing an order in the opaque market if he will not collect enough positive signals in terms of visible order imbalance and hence relinquish profitable trading opportunity. This adverse scenario occurs if the threshold h^* is not attained with a certain time span T given that the demand is high. The probability of $h < h^*$ for any $0 < T < \infty$ is

$$P_N(T, h^*) = 1 - h^* \left(\frac{1-\theta}{\theta}\right)^{-h^*/2} \int_0^T \frac{e^{-\lambda t}}{t} \cdot \mathcal{I}_{h^*}\left(2\lambda t \sqrt{\theta(1-\theta)}\right) dt, \tag{3.8}$$

where $\mathcal{I}_h(\cdot)$ is a modified Bessel function. In this optimal timing model the trader eventually places his order. Therefore, the further away the expiry T, the less likely the trader to forego trade execution at a favourable price A. In order to assess which time constraints are plausible and which are too stringent, it is appropriate to calculate the expected time of order placement:

$$E[\tau^*] = h^* \left(\frac{1-\theta}{\theta}\right)^{-h^*/2} \int_0^T e^{-\lambda t} \cdot \mathcal{I}_{h^*}\left(2\lambda t \sqrt{\theta(1-\theta)}\right) dt. \tag{3.9}$$

Furthermore, assuming that the demand in the opaque market is solid, there is an expression for the conditional expected submission time:

$$E[\tau^*_{\gamma=1}] = h^* \frac{\theta^{h^*} + \xi(1-\theta)^{h^*}}{(1+\xi)\sqrt{(\theta(1-\theta))^{h^*}}} \int_0^T e^{-\lambda t} \cdot \mathcal{I}_{h^*}\left(2\lambda t \sqrt{\theta(1-\theta)}\right) dt. \tag{3.10}$$

Hence, imposing a deadline on the trader can force him to quit the opaque market prematurely. Worse still, in the absence of a deadline factor hastening the trader to make a decision, he might get overconfident about the market demand simply because his ability to read signals is weak. I evaluate whether the chances of this misjudgement are trivial or not by calculating the probability that the trader submits his order while the true demand is low. This probability equals the ex ante probability of reaching the threshold h^* despite insufficient volume in the book:[11]

$$P_Y(h_0, h^*) = \left(\frac{\theta}{1-\theta}\right)^{h_0 - h^*}. \tag{3.11}$$

The probability $P_Y(h_0, h^*)$ decreases when transparency parameter θ improves. This probability coincides with the probability of converting to a market order at the price B, so the probability that the order will trade at $A = B + s$ is $1 - \left(\frac{\theta}{1-\theta}\right)^{h_0 - h^*}$. An alternative interpretation of a premature

[10]The derivations required for the following error analysis are collected in Appendix C.

[11]The derivation of this probability is presented in Delaney and Thijssen (2011).

order placement probability $P_Y(h_0, h^*)$ on the aggregate level is that even if all traders intended to provide liquidity to opaque markets, $P_Y(h_0, h^*)$ is a fraction of traders who end up consuming liquidity instead.

3.6 Numerical Example

To illustrate the analytical results obtained in previous sections I examine these properties through a numerical example with the baseline parameters summarised in Table 3.2. The sunk costs of placing the order are £92, the expected market price of the asset is £90, the limit price is £100[12] and cash flows are discounted with factor $r = 1\%$. On average two orders appear in the book per time period, and the probability of a new order is 70%. The trader has no insider knowledge of the market: he observes no order imbalance and assumes initially equal chances of a high and low demand.

Table 3.2: Baseline parameters of the opaque market.

Parameter	Value
submission fee	$C = £92$
bid price	$B = £90$
spread	$s = £10$
order arrival rate	$\lambda = 2$
incoming order type	$\gamma = \{0 \vee 1\}$
probability of a new submission	$\theta = 0.7$
discount rate	$r = 0.01$
prior belief	$p_0 = 0.5$
order imbalance	$h_0 = 0$

In this benchmark case the transparency is reasonably high with 70% probability of a new order. The threshold probability belief is $p^* = 0.9218$, which implies that the trader wishes to get more convinced before submitting his order and the required visible order imbalance is $h^* = 3$ (or $h(p^*) \approx 2.9112$). The ex ante probability of converting to a market order is almost negligible at 7.87 %. However, the probability that the trader does not place his order and misses a profitable execution opportunity is contingent upon his trading horizon. Given that the trader requires 3 net positive signals in terms of visible order imbalance to submit an order to the opaque market, if he allocates 2 periods to reach a decision, expecting to observe 4 orders during this time, then he makes a wrong choice with probability 64.13%.

Three limiting cases help to locate the baseline case on the spectrum of transparency regimes. First, it is easy to see that when $\theta = 0.5$ the arrival of buy and sell order is equally likely, then the

[12]The existence of a solution to the optimal threshold policy requires that $B + s > C > B$.

threshold

$$p^*(\theta = 0.5) = \frac{C - B}{s} = 0.2000$$

is independent of market parameters other than prices, and order imbalance h^* is any real number. After the trader's belief in a good market reaches 20%, the utility $U(p^*)$ transfers to a region of positive values given that it equals exactly zero at the optimal threshold and increases monotonically in p. Second, $\theta = 0$ is the case of a dark market when the entire book depth is unobservable. In these conditions it is impossible for the trader to assess market depth and thus make an opinion about the asset demand. In this dark market the trader has a slim chance to guess the true state correctly and simply minimises his losses. The threshold is relatively low because the trader accrues confidence too slowly:

$$p^*(\theta = 0) = \frac{r(C - B)}{(r + \lambda)s - \lambda(C - B)} = 0.0012.$$

Lastly, $\theta = 1$ represents a genuinely transparent electronic market where the threshold is higher as compared to two cases above:

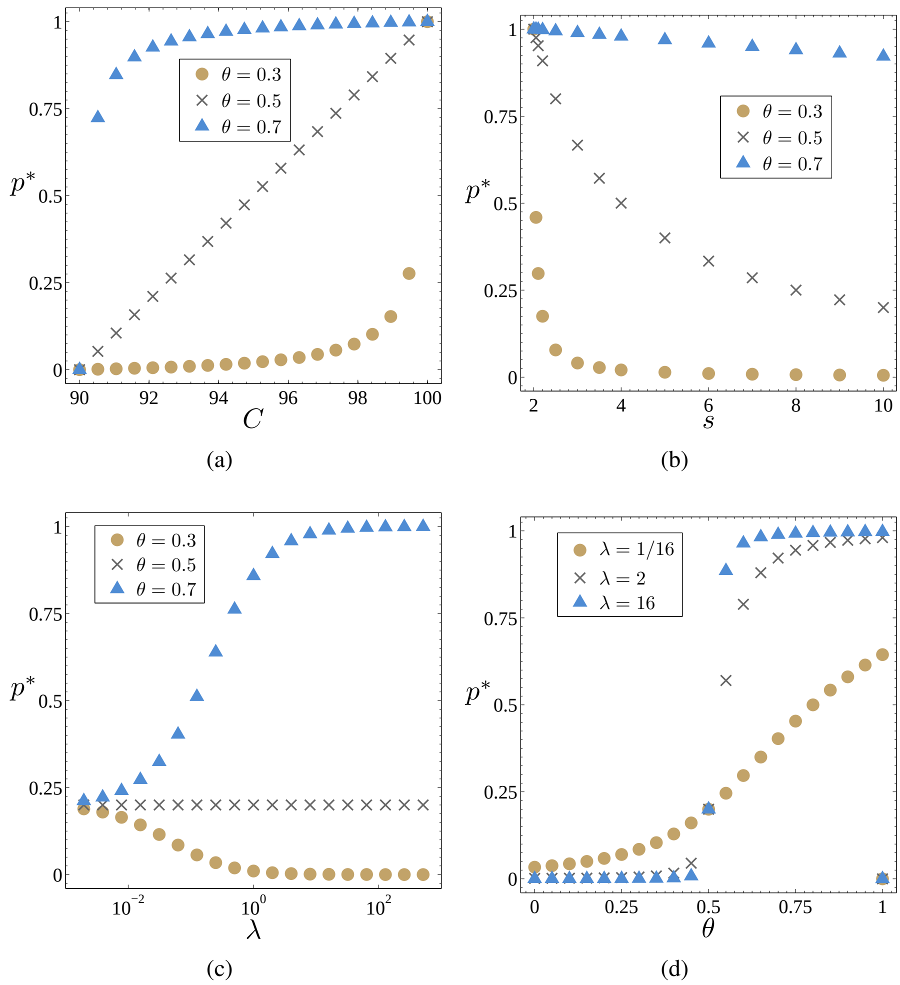
$$p^*(\theta = 1) = \frac{(\lambda + r)(C - B)}{rs + \lambda(C - B)} = 0.9805.$$

Although in a dark market the trader requires a very low threshold to place his order, while under perfect transparency – almost certainty of a high demand, in these polar settings the sufficient imbalance is only one order, that is $h^* = 1$. This is an interesting implication: the trader seems to engage in riskier strategies in an opaque market since he is satisfied with a weaker belief to commit to trade.

3.6.1 Sensitivity Analysis

In this section I explore the comparative static effects of various market conditions on the optimal threshold policy. Overall, I observe nonlinear dependencies which are consistent with the theoretical implications presented in Section 3.4 and confirm certain empirical findings. First, I look at the impact of a single factor on the optimal threshold in isolation (Figure 3.4). Figure 3.4(a) depicts that the positive relationship between the fees C and the threshold belief in a high demand p^* that is consistent with Proposition 3.4.2: higher costs lead to bigger losses in the adverse scenario, thereby the trader is more cautious about his order placement in this case.

Figure 3.4(b) reveals that the more aggressive the limit price A relative to expected market valuation of the asset B, i.e. the wider the bid-ask spread, the lower is the threshold p^*. In other words, when the gains from trading via a limit order are substantial, the trader will place his sell order sooner. This result relates to the notorious gravitational pull effect, which suggests that when the bid-ask spread tightens, the gains from limit orders diminish pushing traders to demand immediacy,

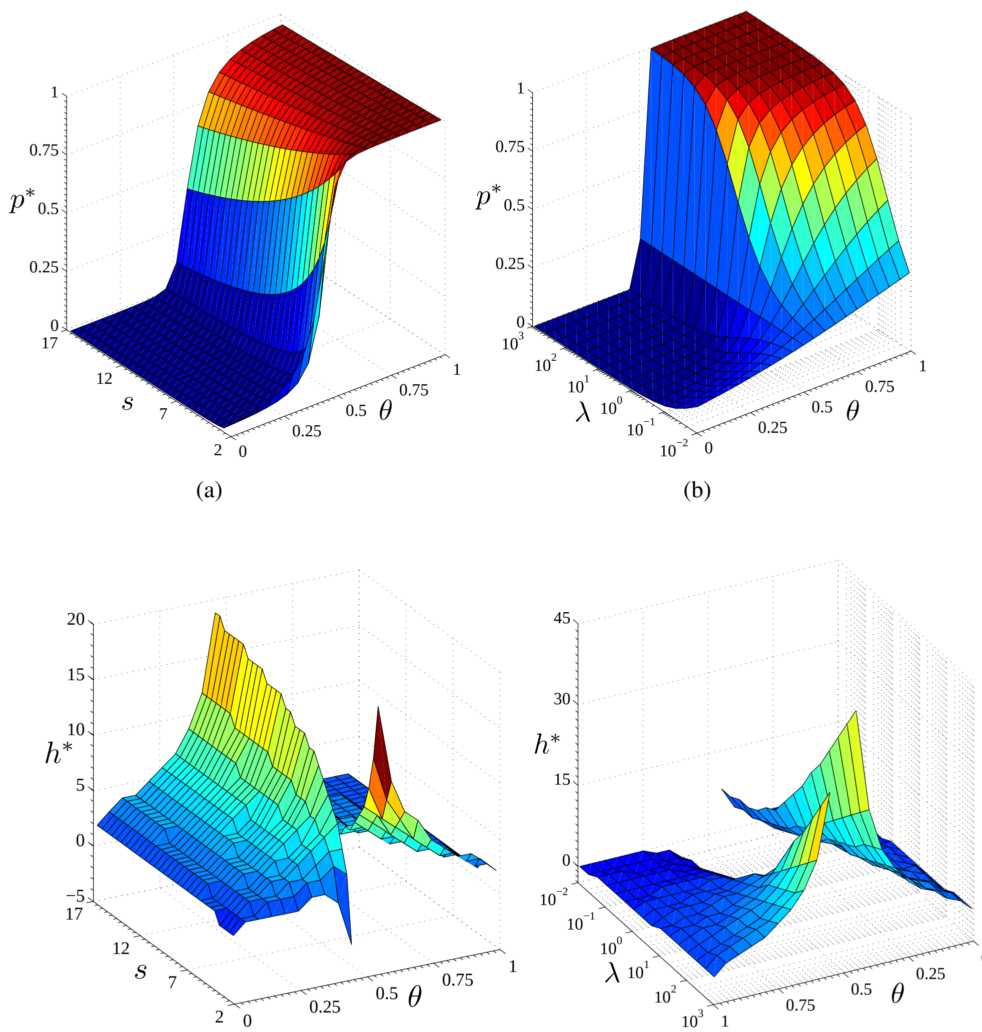


(a) (b) (c) (d)

Figure 3.4: The effects of market conditions on the probability threshold: (a) the submission fee, (b) the spread, (c) liquidity, and (d) transparency. The results are given for the parameters in Table 3.2.

and vice versa. Figure 3.5(a) illustrates that this relationship holds for all transparency levels. At the same time, visible order imbalance required to persuade the trader to accept the non-execution risk of a limit order is not monotonic in the bid-ask spread. According to Figure 3.5(c), if the market is fairly transparent, then h^* decreases with the spread implying that passive strategy is very valuable. This tendency is supported by empirical observations that limit orders are preferred at times when spreads are wide. In a darker market, however, uncertainty is higher, and widening of the bid-ask spread exacerbates downside risks of a limit order strategy, therefore, the trader needs stronger positive evidence to place the order.

The intensity of the order flow usually reflects liquidity strength in a market, which undoubtedly belongs to the list of characteristics that investors evaluate when contemplating whether to join a market. Figure 3.4(c) depicts that depending on the level of transparency, the order arrival rate



(a) (b) (c) (d)

Figure 3.5: Joint effects of two factors on the probability threshold: (a) transparency and spread, (b) transparency and liquidity, and the corresponding joint effects on the visible imbalance threshold: (c) transparency and spread, (d) transparency and liquidity. The results are given for the parameters in Table 3.2.

can have one of the two opposing effects on the threshold p^*. If transparency is high, the more information arrives per unit of time, the more patient the trader in the sense that he will wait longer to pay the fee and resolve uncertainty. If transparency is below 50%, higher liquidity prompts the trader to act faster. In real markets, persistent order flow can be interpreted as a sign of good execution in the market, so traders might mimic each others strategies. In Figure 3.4(d) for any given liquidity level, the higher transparency, the more drastic asymmetry towards the inflow of buyers is required to submit a passive sell order. The compound effect of liquidity λ and transparency θ in Figure 3.5(b) and Figure 3.5(d) suggests that higher liquidity reinforces the consequences of a reduced transparency regime in this double auction market in such a way that if transparency is reasonably high, then the trader prefers to postpone his order submission and collect more signals. In contrast, in a darker market higher liquidity motivates the trader to submit his limit order with a lower confidence in a high market demand, although he raises the imbalance threshold all the same.

Bessembinder et al. (2009) find that hidden orders are more likely when order flow is low, spreads are wide and the competition on the same side of the book is small. Their study suggests that traders hide more when their orders are aggressive. Also, hidden orders are more likely for less liquid firms, essentially confirming that the permission of undisclosed limit orders fulfills its mission in attracting liquidity. Bessembinder et al. (2009) estimate that on average across firms 44% of order flow in volume is hidden which corresponds to the case of $\theta > 0.5$ in my model and is consistent with the result that p^* increases with λ. In other words, for higher liquidity levels the trader is inclined to postpone his order submission. However, I also find that the relationship between p^* and λ is reverse in case of lower transparency levels θ, which is explained by the fact that if the visible liquidity is sufficiently high, the trader expects total market depth to be large and submits his order sooner before this volume is exhausted. According to Aitken et al. (2001), the use of hidden orders increases with the volatility of the stock price and the order value. On the contrary, hidden orders are used less the larger is the tick size, which is reasonable since front-running becomes more costly in these circumstances. Though my model does not account for the stochastic nature of asset prices, and hence the tick size, I observe a similar dependence between the total hidden depth and the optimal timing of order submission: the lower is the degree of market transparency θ, implying a larger associated undisclosed volume, the lower the threshold belief p^* that the trader has to achieve to submit his order. The positive relationship between transparency θ and the optimal threshold belief p^* plotted in Figure 3.4(d) holds for various levels of market liquidity λ.

3.6.2 Error Analysis

As was briefly mentioned in the model outline in Section 3.3, the level of market transparency governs the limiting behaviour of the learning trader when the number of accumulated signals goes to infinity. If the degree of transparency is fair ($\theta > 0.5$), then $\lim_{h\to+\infty}(p_t) = 1$ and $\lim_{h\to-\infty}(p_t) = 0$. This implies that the more positive signals the agent collects, the more likely he is to commit to trade in this market. If transparency is poor ($\theta < 0.5$), then $\lim_{h\to+\infty}(p_t) = 0$ and $\lim_{h\to-\infty}(p_t) = 1$. This case is associated with a contrarian behaviour: if too many buys arrive to the market, the agent believes that the hidden depth is too thick, and although the aggregate market demand might be substantial, the abundance of iceberg orders will prevent the price from moving toward his preferred limit $B+s$. So, in these circumstances the agent will not place his order. Conversely, if mainly sell orders arrive, the agent concludes that there is a good chance of execution and follows the crowd, hoping that these old iceberg orders will be exhausted soon. The probability threshold p^* exists and is well-defined for both regimes $\theta > 0.5$ and $\theta < 0.5$.

Recall the probabilities of misjudging the true market demand in Section 3.5. Consider first the situation where the trader is obliged to close his position with a time constraint T and the true demand for the asset is high. There is a positive probability $P_N(T, h^*)$ that the trader misses the opportunity to trade in the opaque market at a favourable price A because even if the demand for the asset is sufficient, the required threshold of visible imbalance h^* might not be reached before expiry. The longer is the trader's horizon T, the slimmer is the probability of missing a more preferable execution at a high price A, as depicted in Figure 3.6(a). In a more transparent market the probability of missing a high demand is declining more abruptly once $T \gg 1$ (the blue bottom curve), whereas in the opaque market with the clarity of signal only 30% (the brown top line) the probability curve is more shallow with a non-trivial chance of mistake even for a patient trader with a horizon $T = 25$ periods. Actually, in a darker case the number of net positive signals required to place the order is twice as high, so for a similar order flow intensity one needs to wait longer simply to collect any signals.

Further, I plot the expected time of order submission for the time-constrained trader as a function of market transparency θ (Figure 3.6(b)). The relationship is non-monotonic and non-continuous due to the discreteness of the order imbalance signals h^*. Figure 3.6(d) depicts clear jumps in the optimal signal threshold h^* as a function of market transparency θ. For certain bundles of liquidity and transparency, this threshold jumps from $\lceil h(p^*) \rceil$ to $\lceil h(p^*) \rceil + 1$. As p^* increases with θ, the signal threshold h^* is bound to increase as well for low transparency and to decrease otherwise. This relationship translates directly into expected times $E[\tau^*]$ and $E[\tau^*_{\gamma=1}]$: when there is an upward jump in h^*, the expected time jumps as well and then continues to decline; with a downward shift in h^* the expected times of order submission diminish. For each one-unit change in the signal threshold the expected time jumps roughly by half a period, since the average time between order arrivals in this opaque market is $1/\lambda = 0.5$. Notably, irrespective of the degree of market transparency, the trader places his order well before the deadline $T = 25$. For instance, if θ is large then the trader's belief in the high demand becomes stronger with fewer good signals. At the same time, if the order flow is rather uninformative (θ is only a little above 50%), then just a slight prevalence of incoming buys is required to believe in a high demand and as a result the trader will submit his order sooner. Conditional upon a strong demand, order submission happens on average twice as fast: if the trader has a deadline of 25 periods to execute his trade, then he submits the order much quicker with the expected time of submission $E[\tau^*] = 3.7013$, and $E[\tau^*_{\gamma=1}] = 1.9963$ conditional on a high actual demand in the baseline case.

Finally, I consider a reverse scenario, which occurs with a probability $P_Y(h_0, h^*)$: the market demand is poor but the agent collects enough positive signals h^* to commit to trade. In line with

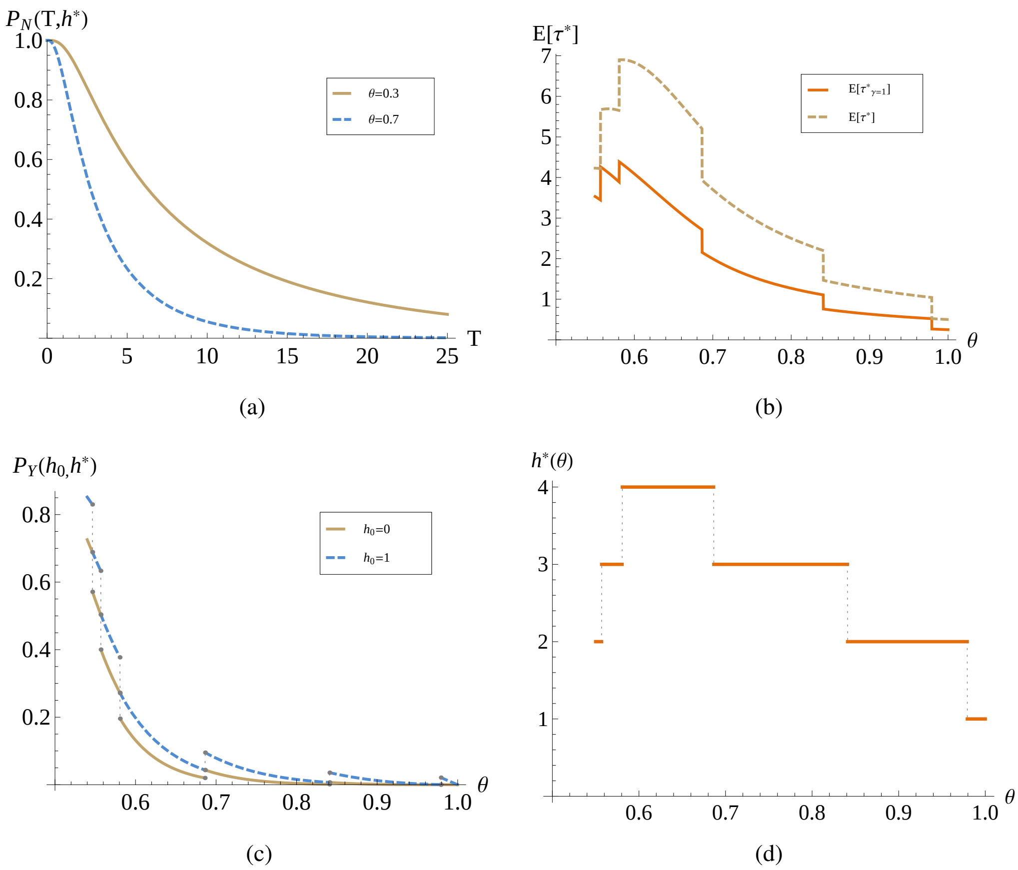


Figure 3.6: (a) The probability of order placement when demand is weak, (b) the expected times of order submission for a trading horizon of 25 periods, (c) the probability of no order placement when the demand is high, and (d) the threshold order imbalance as a function of transparency. The results are given for the parameters in Table 3.2.

intuition, the more transparent the market is, the smaller the probability of this mistake as Figure 3.6(c) vividly demonstrates. The brown curve corresponds here to the case when the trader does not have any preliminary insight about the demand and observes no order imbalance at the start, exactly as in the benchmark case, whereas the blue curve on top corresponds to the case when the agent has initial optimistic bias, which certainly makes him more prone to errors in this model. Again, the jumps in $P_Y(h_0, h^*)$ result from the discontinuities in h^* (Figure 3.6(d)).

3.7 Conclusions

In this chapter I establish a framework that delineates the decision making process of a risk-neutral trader in a market with hidden liquidity. Through a real option approach I analyse how various market parameters, such as execution prices, submission fees, market liquidity and transparency mould the trader's order placement timing. A risk-neutral trader, who is willing to sell his assets, estimates the demand using order flow information streams. The true market depth, which determines the transaction price, is not observed by agents operating in this opaque market. I provide an analytical solution in the form of a threshold policy, such that once the trader's posterior belief in a high demand exceeds this threshold, he immediately submits a limit order to the market. This stylised theoretical model appears to be one of the first attempts in the literature to challenge investors' incentives to participate in opaque trading environments.

The model implies that the trader requires less conviction in a high demand before placing an order once the submission fees diminish or the value of the asset appreciates. This result stems from the utility-maximising behaviour of the trader. More importantly, the threshold belief increases with market transparency. This outcome is contingent upon multiple sources of uncertainty in this framework. Given this, there are intervals of the transparency parameter values when an increase in the probability threshold is associated with fewer positive signals required to convince the trader of a strong demand. Therefore, lower transparency motivates traders to submit their orders to the limit order book quicker; whereas a higher degree of market transparency makes the same agents eager to postpone order placement until their posterior belief in good market conditions becomes more solid. In this manner the model establishes some theoretical foundation that justifies how market opacity can encourage liquidity provision, which, in turn, falls in line with empirical evidence on the expansion and growing popularity of this type of trading venues. In addition, I show that market liquidity, measured by incoming order flow intensity, accentuates the effect of transparency on the optimal threshold belief: when the degree of transparency is above 50%, the threshold increases with liquidity, while in a more opaque market the trader prefers to wait. This is reasonable since in the latter case the trader contemplates that there is a substantial hidden volume in the market coupled with the visible part, while in the former case the fact that most of the market depth is observable makes him more cautious.

Certainly, under any market conditions encompassed by this model, traders are not insured against misjudging the market. The error analysis reveals that traders are less inclined to overvalue the asset demand once any prior biases are removed. Time pressure, however, can pose a significant risk of missing a profitable execution opportunity. Such risk remains substantial in the opaque market even for traders with long time horizons.

The present framework assumes that once the order is placed, it will be fully executed at its limit price or at the market if the demand for the asset is weak. One possible extension of the model is to introduce a scenario with partial execution at the preferred price and monitor how the optimal policy will alter. Furthermore, it would be interesting to develop a version of this model with a distinct stochastic process for the dynamics of order arrival to account for the empirical fact that in opaque markets liquidity often comes sporadically and in clusters (Lehalle et al. (2012, p.47)). Lastly, the model would potentially benefit from incorporating the stochastic nature of the underlying asset price and linking it to the incoming order flow, as well as the possible revision of the limit price by the trader. However, this would probably be achieved at the expense of tractability, and it is likely that an analytical solution could not be obtained and one would need to resort to numerical methods.

Appendix A Proof of Proposition 3.4.1

The critical value of the conditional belief in a high demand for the asset, denoted $p^* = p(h^*)$, is the point such that the trader is indifferent between execution at a limit price or at the market. That is, if $p_t > p^*$, the trader is confident that there will be sufficient demand to sell high. Conversely, if $p_t < p^*$, the trader is not confident enough in the available demand and waits for more information to arrive. For the simplicity of notation I omit in the proof the time index and write instead $h = h_t$ hereafter. To derive the optimal threshold h^* it suffices to split the state space into three possible situations. The first one when the net number of positive signals already exceeds the required threshold ($h \geq h^*$); the second, when the net number of positive signals is much lower than the requirement ($h < h^* - 1$); and the third, when the net number of positive signals is in the optimal vicinity, so that the optimum can be reached at the next step ($h^* - 1 \leq h < h^*$).

Case 1: $h \geq h^*$

In case when current order imbalance exceeds the optimal threshold h^*, the trader immediately submits his order. So his payoff is

$$G(h) = p(h) \cdot s + B - C. \tag{3.12}$$

Consider now the interpretation that the trader attaches to every change in the observable order imbalance in the opaque market. Given the trader's interpretations of the order flow signals, he perceives the visible order imbalance as follows:

$$dh_t = \begin{cases} 1 & \text{with probability} \quad \left(\theta \cdot I_{\{\gamma=1\}} + (1-\theta) \cdot I_{\{\gamma=0\}}\right) \lambda dt \\ 0 & \text{with probability} \quad 1 - \lambda dt \\ -1 & \text{with probability} \quad \left(\theta \cdot I_{\{\gamma=0\}} + (1-\theta) \cdot I_{\{\gamma=1\}}\right) \lambda dt \end{cases} \tag{3.13}$$

Indeed, given the intensity of the order flow is λ, the book remains unchanged with the probability $1 - \lambda dt$ at each instant. The probability of an order arrival is λdt, which can be any of the three order types within buy and sell category. When a buy order appears, there are two possible scenarios that lead to this event. First, a completely new buy order is submitted to the book with probability θ. This can be either a new passive buy order added to the book, with or without a hidden chunk, or a market buy that reduces the number of competitors on the sell side, thereby, the total market depth has changed and it is indicative of a strong demand, $\gamma = 1$. Alternatively, an existing buy iceberg order is renewed, triggered by a sell market order that was placed at the previous time step, with probability $1 - \theta$ and the visible buy depth increases. In this case the overall quantity of the order volume recorded in the opaque market does not alter but only the extra volume available on the buy side is communicated to market participants, while true demand for the asset might be poor, $\gamma = 0$. Similarly, a new sell order of any type is submitted with probability θ given that the demand is insufficient, $\gamma = 0$, and $1 - \theta$ is the chance that an old sell iceberg is refreshed while the cumulative market demand is high, $\gamma = 1$.

Case 2: $h < h^* - 1$

In this case, even if the next order to arrive is a buy, visible order imbalance is not going to reach the optimal threshold. Let the value of the trader's strategy be equal V_1, then at the next order arrival it must satisfy Bellman equation:

$$V_1 = \frac{E[dV_1(h)]}{dt}. \tag{3.14}$$

The value of the strategy at any point in time before order submission must be equal the discounted value at the infinite horizon, or $V_1(h) = V_1/r$. On the other hand, the expected change in the value function with respect to order imbalance dynamics given in (3.13) is

$$\frac{E[dV_1(h)]}{dt} = \begin{pmatrix} + V_1(h+1) \cdot (\theta p(h) + (1-\theta)(1-p(h)))\lambda dt \\ + V_1(h) \cdot (1 - \lambda dt) \\ + V_1(h-1) \cdot (\theta(1-p(h)) + (1-\theta)p(h))\lambda dt \\ - V_1(h) \end{pmatrix} / dt,$$

or

$$\begin{aligned} \frac{E[dV_1(h)]}{dt} &= \lambda V_1(h+1) \frac{\theta^{h+1} + \xi(1-\theta)^{h+1}}{\theta^h + \xi(1-\theta)^h} \\ &- \lambda V_1(h) \\ &+ \lambda\theta(1-\theta)V_1(h-1) \frac{\theta^{h-1} + \xi(1-\theta)^{h-1}}{\theta^h + \xi(1-\theta)^h}. \end{aligned}$$

Equating the two expressions in (3.14) and rearranging, I obtain:

$$\begin{aligned} (\lambda + r)V_1(h) \cdot (\theta^h + \xi(1-\theta)^h) &= \lambda V_1(h+1) \cdot (\theta^{h+1} + \xi(1-\theta)^{h+1}) \\ &+ \lambda\theta(1-\theta)V_1(h-1) \cdot (\theta^{h-1} + \xi(1-\theta)^{h-1}). \end{aligned} \tag{3.15}$$

Further, I denote a function $F(h) := V_1(h) \cdot (\theta^h + \xi(1-\theta)^h)$ and transform the equation (3.16) to arrive at the equality

$$\lambda F(h+1) - (\lambda + r)F(h) + \lambda\theta(1-\theta)F(h-1) = 0. \tag{3.16}$$

The ODE (3.16) admits a solution of the form $F(h) = A \cdot x^h$, where A =const and x are the roots of the second order polynomial

$$\Psi(x) = \lambda x^2 - (\lambda + r)x + \lambda\theta(1-\theta). \tag{3.17}$$

The discriminant is $D = (\lambda+r)^2 - 4\lambda^2\theta(1-\theta) = r^2 + 2r\lambda + \lambda^2(1-2\theta)^2$ is non-negative $\forall\theta \in \mathbb{R}$ given both λ and r are positive values. Therefore, both roots of the polynomial $\Psi(x)$ are real numbers: $x_{1,2} = \frac{(\lambda+r)\pm\sqrt{r^2+2r\lambda+\lambda^2(1-2\theta)^2}}{2\lambda} \in \mathbb{R}$.

In case the number of sellers signals is very large, $\lim_{h\to-\infty} V_1(h) = 0$, it must hold that $A_2 = 0$, so the solution is

$$V_1(h) = \frac{A_1 x_1^h}{\theta^h + \xi(1-\theta)^h}. \tag{3.18}$$

Case 3: $h^* - 1 \leq h < h^*$

In this situation, if an additional buy order appears at the next instant, then it becomes optimal to submit a limit order. Again, denote the current value of the strategy by $V_2(h)$, then $V_2(h) = V_2/r$ and another Bellman equation must be satisfied:

$$V_2 = \frac{E[dV_2(h)]}{dt}. \tag{3.19}$$

The expected change in the value function is

$$\frac{E[dV_2(h)]}{dt} = \begin{pmatrix} + G(h+1)\cdot(\theta p(h) + (1-\theta)(1-p(h)))\lambda dt \\ + V_2(h)\cdot(1-\lambda dt) \\ + V_1(h-1)\cdot(\theta(1-p(h)) + (1-\theta)p(h))\lambda dt \\ - V_2(h) \end{pmatrix} /dt,$$

which reduces to

$$\begin{aligned} \frac{E[dV_2(h)]}{dt} &= \lambda\left(p(h+1)s + B - C\right)\frac{\theta^{h+1} + \xi(1-\theta)^{h+1}}{\theta^h + \xi(1-\theta)^h} \\ &- \lambda V_2(h) \\ &+ \lambda\theta(1-\theta)x_1^{h-1}\frac{A_1}{\theta^{h-1} + \xi(1-\theta)^{h-1}}\frac{\theta^{h-1} + \xi(1-\theta)^{h-1}}{\theta^h + \xi(1-\theta)^h}. \end{aligned}$$

Thus, equation (3.19) comes to

$$\begin{aligned} (\lambda + r)V_2(h)\cdot(\theta^h + \xi(1-\theta)^h) &= \lambda\Big((B+s-C)\theta^{h+1} \\ &+ (B-C)\xi(1-\theta)^{h+1} + \theta(1-\theta)x_1^{h-1}A_1\Big). \end{aligned} \tag{3.20}$$

In order to establish a unique solution two additional criteria have to be satisfied: value matching and continuity conditions. These coditions reconcile functions $G(h)$, $V_1(h)$ and $V_2(h)$ at switching points and determine the values of the constants in the solution.

1. Value matching condition: $V_2(h^*) = G(h^*)$

Equate the right-hand side term in equation (3.21) and the expression for $G(\cdot)$ in (3.12) multiplied by $(\lambda + r)(\theta^h + \xi(1-\theta)^h)$ and obtain

$$\begin{aligned}\lambda\left[(B+s-C)\theta^{h^*+1} + (B-C)\xi(1-\theta)^{h^*+1} + \theta(1-\theta)A_1 x_1^{h^*-1}\right] = \\ (\lambda + r)\left[(B+s-C)\theta^{h^*} + (B-C)\xi(1-\theta)^{h^*}\right].\end{aligned} \tag{3.21}$$

From here it follows immediately that the constant A_1 is

$$A_1 = \frac{(r+(1-\theta)\lambda)(B+s-C)\theta^{h^*} + (r+\lambda\theta)(B-C)\xi(1-\theta)^{h^*}}{\lambda\theta(1-\theta)x_1^{h^*-1}}. \tag{3.22}$$

2. Continuity condition: $V_1(h^*-1) = V_2(h^*-1)$

Substituting the value of A_1 in (4.33) into the expressions for values V_1 and V_2, I get:

$$\begin{aligned} V_1(h^*-1) &= \frac{x_1^{h^*-1}A_1}{\theta^{h^*-1} + \xi(1-\theta)^{h^*-1}} \\ &= \frac{1}{\lambda\theta(1-\theta)} \cdot \frac{(r+(1-\theta)\lambda)(B+s-C)\theta^{h^*} + (r+\lambda\theta)(B-C)\xi(1-\theta)^{h^*}}{\theta^{h^*-1} + \xi(1-\theta)^{h^*-1}}, \end{aligned}$$

$$\begin{aligned} V_2(h^*-1) &= \frac{\lambda}{\lambda + r} \cdot \frac{(B+s-C)\theta^{h^*} + (B-C)\xi(1-\theta)^{h^*} + A_1\theta(1-\theta)x_1^{h^*-2}}{\theta^{h^*-1} + \xi(1-\theta)^{h^*-1}} \\ &= \frac{1}{x_1(\lambda + r)} \cdot \frac{(r+(1-\theta+x_1)\lambda)(B+s-C)\theta^{h^*} + (r+(\theta+x_1)\lambda)(B-C)\xi(1-\theta)^{h^*}}{\theta^{h^*-1} + \xi(1-\theta)^{h^*-1}}. \end{aligned}$$

I multiply each expression $V_1(h^*-1)$ and $V_2(h^*-1)$ by $x_1\lambda\theta(1-\theta)(\lambda + r)\frac{\theta^{h^*-1}+\xi(1-\theta)^{h^*-1}}{\theta^{h^*}+\xi(1-\theta)^{h^*}}$ and equate them to obtain

$$\begin{aligned} &p(h^*) \cdot (B+s-C)\left[(r+(1-\theta+x_1)\lambda) - x_1(\lambda+r)(r+(1-\theta)\lambda)\right] \\ &= (1-p(h^*)) \cdot (B-C)\left[x_1(\lambda+r)(r+\lambda\theta) - \lambda\theta(1-\theta)(r+(\theta+x_1)\lambda)\right]. \end{aligned}$$

Solving in terms of $p^* := p(h^*)$ gives

$$p^* = \left[\left(\frac{s}{C-B} - 1\right)\Pi + 1\right]^{-1}, \tag{3.23}$$

where

$$\Pi = \frac{\lambda\theta(1-\theta)(r+(1-\theta+x_1)\lambda) - x_1(\lambda+r)(r+(1-\theta)\lambda)}{\lambda\theta(1-\theta)(r+(\theta+x_1)\lambda) - x_1(\lambda+r)(r+\theta\lambda)},$$

and $x_1 > \theta$ is the larger real root of the quadratic polynomial

$$\Psi(x) \equiv \lambda x^2 - (\lambda + r)x + \lambda\theta(1-\theta).$$

Given that all model parameters are non-negative and $x_1 = (r+\lambda+\sqrt{r^2+2r\lambda+\lambda^2(1-2\theta)^2})/2\lambda$ I simply rewrite the expression for the coefficient Π in a more economical form:

$$\Pi = \frac{r + (1-2\theta)\left(\lambda(1-2\theta) + \sqrt{r^2+2r\lambda+\lambda^2(1-2\theta)^2}\right)}{2r(1-\theta)}. \tag{3.24}$$

Appendix B Proof of Proposition 3.4.2–3.4.5

The sensitivity of the optimal threshold p^* with respect to market characteristics is obtained through partial derivatives. Also, in order to guarantee that p^* is a well-defined probability it must hold that $B+s > C > B$. Given that the coefficient $\Pi \geq 0$, it follows that the threshold p^* depends negatively on the spread s and the bid price B, and positively on costs C:

$$\frac{\partial p^*}{\partial s} = -(p^*)^2 \cdot \frac{\Pi}{C-B} < 0, \tag{3.25}$$

$$\frac{\partial p^*}{\partial B} = -(p^*)^2 \cdot \frac{\Pi s}{(C-B)^2} < 0 \tag{3.26}$$

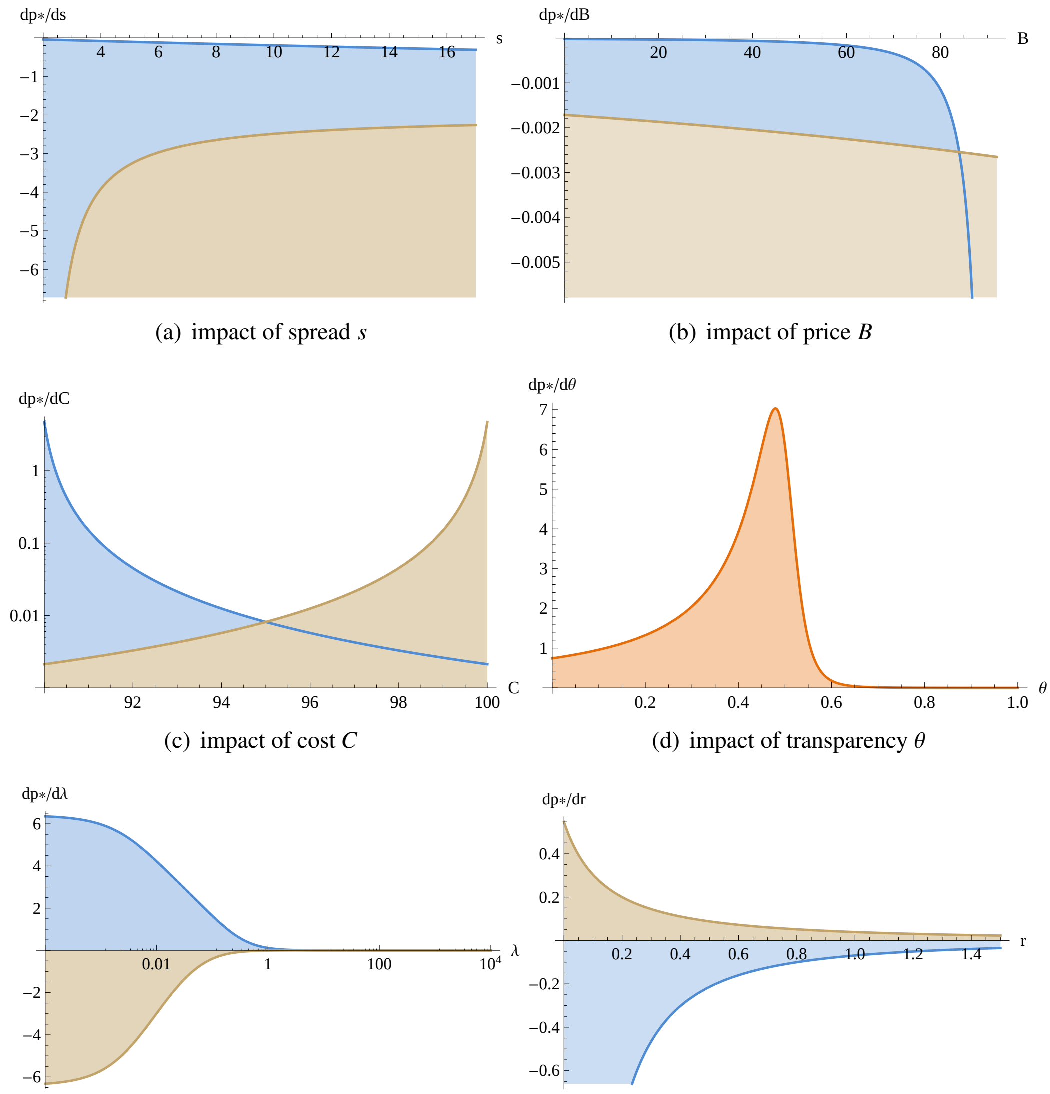
and

$$\frac{\partial p^*}{\partial C} = (p^*)^2 \cdot \frac{\Pi s}{(C-B)^2} > 0. \tag{3.27}$$

These relationships are illustrated in Figure 3.7(a)-3.7(c) for the baseline values of parameters (Table 3.2). Further, I find the derivatives using the implicit function theorem.

Theorem 3.7.1 (Implicit Function Theorem)**.** *Let the function $F : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ have a zero in (x_0, y_0). If $F(x, y)$ is continuous in the vicinity of (x_0, y_0) and strictly monotonic in y for any fixed x, then there exists a continuous two-dimensional interval $\mathbb{I} = \mathbb{I}_x \times \mathbb{I}_y$ in the vicinity of (x_0, y_0) and a continuous function $f : \mathbb{I}_x \to \mathbb{I}_y$ such that for any point $(x, y) \in \mathbb{I}$ it holds:*

$$F(x, y) = 0 \quad \Leftrightarrow \quad y = f(x). \tag{3.28}$$



(a) impact of spread s

(b) impact of price B

(c) impact of cost C

(d) impact of transparency θ

(e) impact of liquidity λ

(f) impact of discount rate r

Figure 3.7: Partial derivatives of the threshold belief with respect to market parameters (blue: $\theta = 0.7$, brown: $\theta = 0.3$).

Corollary 3.7.1. *Suppose that $F(x, y)$ is continuously differentiable, and denote its partial derivative with respect to x by F'_x and the partial derivative with respect to y by F'_y. The partial derivative of the function $f(x)$ is expressed as*

$$f'(x) = -\frac{F'_x(x, f(x))}{F'_y(x, f(x))}. \tag{3.29}$$

Therefore, applying the property (3.29) it follows that the derivative of the optimal threshold p^*

with respect to any factor y is given by

$$\frac{\partial p^*}{\partial y} = \frac{\partial p^*}{\partial \Pi} \cdot \frac{\partial \Pi}{\partial y}. \tag{3.30}$$

First, the partial derivative of threshold p^* with respect to coefficient Π equals:

$$\frac{\partial p^*}{\partial \Pi} = -(p^*)^2 \cdot \left(\frac{s}{C-B} - 1\right) < 0. \tag{3.31}$$

Second, I take the derivative of $\Pi(\lambda, \theta, r)$ with respect to each of the variables λ, θ and r:

$$\begin{aligned} \frac{\partial \Pi}{\partial \theta} &= \frac{-r^2 + r\left(-2\lambda + \sqrt{r^2 + 2r\lambda + \lambda^2(1-2\theta)^2}\right)}{2r(1-\theta)^2\sqrt{r^2 + 2r\lambda + \lambda^2(1-2\theta)^2}} \\ &- \frac{\left(3 - 8\theta + 4\theta^2\right)\lambda\left((1-2\theta)\lambda + \sqrt{r^2 + 2r\lambda + \lambda^2(1-2\theta)^2}\right)}{2r(1-\theta)^2\sqrt{r^2 + 2r\lambda + \lambda^2(1-2\theta)^2}} \end{aligned} \tag{3.32}$$

$$\frac{\partial \Pi}{\partial \lambda} = \frac{(1-2\theta)}{\sqrt{r^2 + 2r\lambda + \lambda^2(1-2\theta)^2}} \cdot \Pi(\lambda, \theta, r), \tag{3.33}$$

and

$$\frac{\partial \Pi}{\partial r} = -\frac{(1-2\theta)\lambda}{r\sqrt{r^2 + 2r\lambda + \lambda^2(1-2\theta)^2}} \cdot \Pi(\lambda, \theta, r). \tag{3.34}$$

Given that $\frac{\partial p^*}{\partial \Pi} < 0$, we infer $\frac{\partial p^*}{\partial \theta} \geq 0$, $\frac{\partial p^*}{\partial \lambda} = \text{sgn}\{\theta - 0.5\}$ and $\frac{\partial p^*}{\partial r} = \text{sgn}\{0.5 - \theta\}$, as illustrated in Figure 3.7(d)-3.7(f).

Appendix C Error Analysis Derivations

In order to derive the probability that the trader does not submit an order while the true state is good, and the corresponding expected times of submission, I use the distribution functions of the first passage time. In a similar stochastic problem, Feller (1971, p.479) provides the density function of the first passage time for the variable that describes the visible order imbalance in my model.

Denote by $f_h(t)$ the unconditional probability density function of the imbalance first passage time:

$$f_h(t) = e^{-\lambda t}\frac{h}{t}\left(\frac{1-\theta}{\theta}\right)^{-h/2} \mathfrak{I}_h\left(2\lambda t\sqrt{\theta(1-\theta)}\right), \tag{3.35}$$

where $\mathfrak{I}_h(\cdot)$ is a modified Bessel function. Then the probability that h^* is not reached before T is calculated as

$$P_N(T, h^*) := P[\forall t \in [0, T] : h_t \leq h^* \big| h_0 = h, \gamma = 1] = 1 - \int_0^T f_h(t)dt. \tag{3.36}$$

I substitute from (3.35) the probability density $f_h(t)|_{h=h^*}$ and arrive at the formula

$$P_N(T,h^*) = 1 - h^* \left(\frac{\theta}{1-\theta}\right)^{h^*/2} \int_0^T \frac{e^{-\lambda t}}{t} \cdot \mathfrak{I}_{h^*}\left(2\lambda t \sqrt{\theta(1-\theta)}\right) dt. \tag{3.37}$$

Next, using the probability density $f_h(t)$ I obtain the expected time of order submission via definition of the mean value $E[x] = \int x \cdot f(x) dx$:

$$E[\tau^*] := E[\tau \leq T | h = h^*] = h^* \left(\frac{1-\theta}{\theta}\right)^{-h^*/2} \int_0^T e^{-\lambda t} \cdot \mathfrak{I}_{h^*}\left(2\lambda t \sqrt{\theta(1-\theta)}\right) dt. \tag{3.38}$$

Lastly, denote by $\tilde{f}_h(t)$ the probability density function of the imbalance first passage time conditional upon a high true demand:

$$\tilde{f}_h(t|\gamma = 1) = e^{-\lambda t} \frac{h}{t} \frac{\theta^h + \xi(1-\theta)^h}{(1+\xi)\sqrt{(\theta(1-\theta))^h}} \mathfrak{I}_h\left(2\lambda t \sqrt{\theta(1-\theta)}\right). \tag{3.39}$$

Therefore, the expected time of submission given a high true demand is determined by the formula

$$E[\tau^*_{\gamma=1}] := E[\tau \leq T | h = h^*, \gamma = 1] = h^* \frac{\theta^{h^*} + \xi(1-\theta)^{h^*}}{(1+\xi)\sqrt{(\theta(1-\theta))^{h^*}}} \int_0^T e^{-\lambda t} \cdot \mathfrak{I}_{h^*}\left(2\lambda t \sqrt{\theta(1-\theta)}\right) dt. \tag{3.40}$$

Chapter 4

Transparency Regimes in a Market with Heterogeneous Trading Rules

4.1 Introduction

As a branch of financial market regulation debates, there is a growing literature on the benefits and pitfalls of reduced market transparency, with a special alacrity regarding iceberg or hidden order trading. The previous chapter was concerned with the incentives structure of an individual investor who contemplates trading in a market with hidden liquidity, departing from a premise that such markets offer, with some probability, better trade execution conditions. The aim of this chapter is to investigate the problem of limited market transparency using an artificial double auction setup, and much of the literature reviewed in the previous chapter remains relevant for the present discussion.

In order to understand the regulators' dilemma in prescribing or banning the use of iceberg orders, consider a following example. Up until the formal introduction of iceberg orders in public limit order markets, brokers established their own automated order processing systems that divided particularly large orders received from the customers into several smaller tranches and then routed these tranches to the market in succession. For instance, if a broker received an order to sell 10,000 stocks at £25, he would split it up into 10 equal orders of 1,000 stocks each and his automated system would repeat the submission of these orders to the public limit order book the moment one of them is fully executed. This strategy permitted to sell the whole amount of stocks preserving the price of £25 per stock. However, this strategy, without being explicitly known to the exchange, would not eliminate the risk of being traded through. Indeed, suppose that the market is trading at £23 − 25 and a seller has an order with a broker for 10,000 shares at £25. If an aggressive buyer arrived to purchase 2,500 stocks at the market, he would fill 1,000 at £25 and buy the rest from the next seller, for example, at £26. As a result both parties would have missed the opportunity of better trade terms: the large seller would have suffered from longer total execution time or only partial execution, and the buyer would have bought at a higher price £26 being unaware of a better price offer. In response to the inefficiency of this large order transmission routine iceberg orders appeared as an innovation that enabled brokers to hold a full customer order in the public order book whilst displaying only a small fraction of it. In the above scenario with an aggressive buyer all 2,500 shares would be purchased at £25, benefiting the buyer by achieving a better price, and the seller by faster execution, and thus increase overall market liquidity.

Beneficial as iceberg orders seem, they entail a certain trade-off: hidden liquidity affects the ability of traders to evaluate the true market conditions, making the social advantages of opacity rather obscure. Recent empirical studies, for instance, Aitken et al. (2001); Anand and Weaver (2004); Bloomfield et al. (2011); Boehmer et al. (2005); Henderhott and Jones (2005), as well as theoretical works of Baruch (2005) and Moinas (2010) explore the subject of market opacity and its influence on market quality. Despite availability of data, there is no consensus in the literature on the role of imperfect transparency: in certain markets reduced transparency widens the spread and deteriorates efficiency (Flood et al., 1999; Henderhott and Jones, 2005), other research reports partially or exactly opposite outcomes (Bloomfield and O'Hara, 1999; Madhavan et al., 2005) or no significant impact (Aitken et al., 2001). While empirical studies seem to diverge in their conclusions partly due to discrepancies in the structures of actual analysed markets, theoretical models suffer from the rigidity of assumptions. This chapter adopts an agent-based modeling approach that surpasses the mentioned limitations of empirical and other theoretical frameworks and facilitates the comparison of various market regimes. Few attempts have been made to date to apply agent-based modeling approach to the problem of market transparency. In his recent paper Yamamoto (2011), as one recent example, argues that the level of quote transparency has little influence on long memory properties of financial data. However, the author concentrates solely on the type of transparency regime when quote visibility in the limit order book is restricted to the five best prices, leaving opaque microstructures with hidden and iceberg orders outside his scope.

In this chapter I contribute to the research on pre-trade market transparency. I examine the consequences that different market depth disclosure policies propagate in an artificial double auction market where agents use the information about volumes in the limit order book to formulate their trading strategies. In particular, I contrast an exogenous transparency regime, whereby the exchange explicitly regulates the number of publicly displayed quotes, to an endogenous transparency regime, where the amount of displayed volume is determined by traders themselves by means of iceberg orders. My analysis involves several dimensions which I synthesise into two broad and partly overlapping research questions.

How is reduced market transparency reflected in order placement?

One crucial aspect of market operations is encoded in the order flow. In an artificial double auction trading environment with moderately sophisticated agents, I distinguish between market orders, aggressive and passive limit orders. Based on this classification I estimate the distribution of orders that are placed to the market conditional upon the preceding order type. Furthermore, I compare the conditional aggressiveness of orders subject to current market conditions and monitor how frequencies of each order type evolve as market transparency diminishes.

What is the impact of lower pre-trade transparency on market quality?
Among the indicators of market quality I define transaction costs measured by the bid-ask spread and the liquidity evaluated by such metrics as price impact and execution delay that occurs after the order became the best price. Moreover, I examine market efficiency and memory properties embedded in asset returns and order book imbalance. Since there is no clear alignment in the empirical research regarding the implications of reduced trade transparency, I expand the literature on the subject and assess the variation of results across different transparency regimes.

The remainder of this chapter proceeds as follows. Section 4.2 presents the design of a market in which traders submit their orders with the clearing mechanism, and defines market specifications. Section 4.3 evaluates the capability of this artificial market with heterogeneous traders to replicate the phenomena that is often encountered in financial markets for different degrees of transparency. Section 4.4 elaborates the analysis of restricted depth information within four distinct market architectures and scrutinises the evolution of market quality characteristics: the size of bid-ask spreads, regularities in the order placement, and market liquidity, assessed by various indicators. Section 4.5 concludes and points out directions for future research.

4.2 Market Setup

4.2.1 The Model

I construct a double auction market for a single non-dividend paying stock where N_A heterogeneous agents trade during a repeated number of rounds, by modifying the prototype model of artificial market of Chiarella et al. (2009). In each time period t a random agent i is called to trade. This agent formulates his order placement strategy by (i) relating the current price of the asset to its fundamental value, (ii) analysing information about the visible market depth, and (iii) assuming some random component in the asset returns. Assuming that the market reverts to trading at the fundamental price of the asset p_t^f in τ^f periods, the trader evaluates in which direction the market is likely to move given the current price p_t. The order imbalance is defined as the signed log difference between the total amount of stocks demanded Q_t^b and offered Q_t^a in the market at time t

$$D_t = \operatorname{sgn}\left(Q_t^a - Q_t^b\right) \cdot \ln\left(1 - \frac{\left|Q_t^a - Q_t^b\right|}{Q_t^a + Q_t^b}\right). \tag{4.1}$$

If the combined volume of orders submitted on the same side as his trading intention is larger than on the opposite side of the market, the agent realises that there is a tight competition, and that the chances to trade within his time horizon are slim, hence he is prone to trade more aggressively. Also, the trader takes into account an aggregated uncertainty factor ξ that is normally distributed with zero mean and σ_ξ^2 variance, $\xi \sim N(0, \sigma_\xi^2)$. Therefore, trader i builds his expectation of the return on the asset that can be achieved within his time horizon τ^i according to the following rule:

$$\hat{r}^i_{t,t+\tau^i} = \frac{1}{\nu_1^i + \nu_2^i + \eta^i} \left[\nu_1^i \frac{\ln(p_t^f / p_t)}{\tau^f} + \nu_2^i \bar{D}_t + \eta^i \xi_t \right], \tag{4.2}$$

where $\bar{D}_t$ is the average order imbalance over the past τ^i periods, and coefficients ν_1^i , ν_2^i and η^i guide fundamentalist, imbalance and noise components of trader's i strategy respectively. The forecasted rate of return $\hat{r}^i_{t,t+\tau^i}$ yields the maximum expected price over a τ^i periods

$$\hat{p}^i_{t+\tau^i} = p_t \exp(\hat{r}^i_{t,t+\tau^i} \tau^i). \tag{4.3}$$

Every trader in this market is risk-averse and holds a portfolio of stocks and cash.[1] At each trading round the trader that was summoned to the market establishes a range of admissible prices. I assume that short selling is prohibited, hence the maximum price p_M at which trader i can place his order equals his expected price $\hat{p}^i_{t+\tau^i}$. The lowest price p_m at which trader i can place an order is subject to his budget constraint since borrowing is not available. In addition, there is a price p^*, a *satisfaction level*, that makes the current portfolio composition optimal: if the current market price of the stock is above this level then the trader is willing to sell, otherwise, the trader intends to buy more stocks. Two propositions below describe the mechanism that guides the trader's response to visible order imbalance, depending on the force of imbalance impact, and to the volatility of past returns on the interval of admissible prices.

Proposition 4.2.1. *Assume that in a market with short selling ban and no borrowing available to the traders the region of admissible buy* $[p_m, p^*]$ *and sell* $[p^*, p_M]$ *prices for a given risk-averse trader is determined by equations (4.23)-(4.25). Then it follows that*

• *both the satisfaction price* p^* *and the minimum price* p_m *are monotonic increasing in the maximum expected price* p_M*;*

• *the sensitivity of the satisfaction price* p^* *to* p_M *is higher than the sensitivity of the minimum price* p_m *to* p_M*.*

Proof. See Appendix B. □

[1] This model bears a close resemblance to the framework described in Chiarella et al. (2009) with the imbalance factor in place of chartist. For this reason I highlight here only key aspects of the model with emphasis on the role of order book imbalance. Interested readers are referred to Appendix A of this chapter for a full description of traders' decision making and order submission routines.

Proposition 4.2.1 implies that if the trader is optimistic and wishes to sell stocks, then bearing in mind the current location of the asset price p_t, he is likely to place a limit order farther in the book and wait till the market reaches this price level. However, if the seller is convinced of price depreciation then he trades at the market. Likewise, buyers that believe in price decline replenish the book, while optimistic buyers demand immediacy instead. Given this, positive order imbalance translates into higher price expectations and, therefore, pushes buyers to submit more aggressive orders and sellers – more passive. Similarly, when sell limit orders prevail in the book and order imbalance is negative, new incoming sellers are more prone to consume liquidity and buyers are more likely to provide liquidity to the market. Hence, impatient traders reinforce the trend in imbalance, and patient agents reverse it.

Proposition 4.2.2. *Assume that in a market with short selling ban and no borrowing available to the traders the region of admissible buy $[p_m, p^*]$ and sell $[p^*, p_M]$ prices for a given risk-averse trader is determined by equations (4.23)-(4.25). Then it follows that*

(1): if the order imbalance towards buyers increases, the trader is more likely to sell the stocks, and vice versa;

(2a): if buyers prevail in the limit order book, the trader with a stronger imbalance impact is more likely to sell the stocks, and vice versa;

(2b): if sellers prevail in the limit order book, the trader with a stronger imbalance impact is more likely to buy the stocks, and vice versa;

(3): if the volatility of returns increases, the trader is more likely to sell the stocks, and vice versa.

Proof. See Appendix C. □

The relationship between order imbalance and traders' propensity to buy or sell the asset suggests that quote transparency intensifies contrarian behaviour among market population. Indeed, if order imbalance is skewed to one side, then the new agent that arrives to trade is inclined to join the opposite side of the market.

The resulting mixture of trend-following and contrarian behaviour that stems from Propositions 4.2.1 and 4.2.2 is rooted to recurring empirical evidence. Ranaldo (2004) discovers that patient traders seek liquidity when the same side of the book is dense and the opposite side is slimmer. Lo and Sapp (2005) document a complementary effect: traders switch to more passive orders if many aggressive traders precede on the same side of the market. In addition, Chiarella et al. (2009) demonstrate that the behaviour induced on noise traders by CARA preferences implies that if the noise traders are optimistic, they submit all types of orders symmetrically around the quotes, whereas pessimistic expectations makes them more likely to buy with limit orders and sell with market orders. As I reveal in the numerical simulations, when traders take market depth into consideration, the book becomes more symmetric and bears closer resemblance to the average book shape observed in real markets.

4.2.2 Simulations Design

In this section I outline the main features of the simulations and provide a summary of numerical parameters. The ultimate aim of the present chapter is to trace the reaction of heterogeneous agents to the availability of information about market depth. In the simulations I juxtapose markets with four alternative transparency regimes: with no information and full information about the quote depth, and then with exogenous and endogenous restrictions on displayed market depth. For convenience, I label these market environments as *Specification 1*, *Specification 2*, *Specification 3* and *Specification 4* respectively, as summarised in Table 4.1.

Table 4.1: The definitions of the four market transparency regimes.

Specification	Name	Description
Specification 1	dark market	fundamentalist and noise impact
Specification 2	transparent market	fundamentalist, noise and order imbalance impact
Specification 3	quasi-transparent market	fundamentalist, noise and order imbalance impact with only five best quotes visible
Specification 4	opaque market	fundamentalist, noise and order imbalance impact and permission to use iceberg orders

I start by comparing the extreme cases of a dark market, where market depth is not publicly revealed and traders have no resources to estimate the imbalance D_t,[2] and a transparent market, where traders are aware of all orders sizes sitting in the book. These two markets serve as benchmarks in my analysis. Naturally, in a transparent market agents observe the depth of an entire limit order book before trading. While in certain markets such an assumption is perfectly viable (e.g. NYSE OpenBook), many other exchanges (e.g. Tokyo Stock Exchange, Euronext Paris) restrict the available information to the five best quotes on each side of the book at any time. The objective that trading venues pursue by imposing this restriction is to attract more traders and avoid price manipulation by placing misleading orders. Therefore, in one of the simulation specifications I consider a quasi-transparent market with an exogenous restriction where traders formulate their strategies according to the same principle as in a transparent market but with an extra condition imposed on their information set: they calculate order imbalance D_t based only on the visible depth of the five best quotes on the buy side and the five best offers on the sell side at time t.

Lastly, suppose that the use of iceberg orders, which allow traders to fix the limit price and leave a fraction of the order size temporarily hidden, is permitted in the market. I introduce a market rule

[2] All agents have a zero weight of imbalance component $\nu_2^i = 0, \forall i$ in equation (4.2).

whereabout any trader may hide their full order size subject to an imposed minimum size.[3] This market layout is referred to as an opaque market throughout the discussion.[4] Any trader in this opaque market can take advantage of an iceberg order to manage his exposure. The algorithm of order submission is similar to the basic model of Section 4.2.1 with an additional step to determine the appropriateness of splitting via an iceberg order. I set the minimum iceberg order peak size equal to the current average order size on the relevant side of the book. Therefore, if agent i was randomly called to trade in round t and picked a price p, so that, for example, $p_m \leq p \leq a_t$, he would pursue a passive buy strategy. This trader then compares his demand for stocks to the average buy order size in the book $\bar{Q}_t$. If the amount of stocks that the trader wishes to purchase is below the mean size of visible buy limit orders, then he submits an ordinary limit buy order; otherwise, the trader uses a soft approach and places an iceberg order at price p with the peak size $\bar{Q}_t(1 + u)$, where $u \sim U(0, 1)$ is a standard uniform random variable. As a result, the depth of the limit order book is only partly observable and traders are forced to use just visible volumes to assess the order imbalance and predict the future asset price.[5]

The summary of the parameter values adopted in numerical simulations of the four described market layouts is given in Table 4.2 below. There is a fixed population of N_A agents in the market, who are initially allocated a random amount of stocks uniformly distributed on the interval $S_0 \in [0, N_S]$ and an amount of cash $C_0 \in [0, N_S p_0]$ with a maximum number of shares N_S per trader and the common risk aversion factor φ. The starting price is $p_0 = £400$ and the price grid is determined by the minimum tick size Δ. The noise term ξ_t is normally distributed with dispersion σ_ξ^2. Agents trade sequentially and in a random order. When called to trade, an agent determines his price interval $[p_m, p_M]$ and draws any price p from that interval with an equal probability. If the trader already has an outstanding order in the book, it is replaced by a new one. Agents do not revise or cancel their orders prior to expiry otherwise. The trading horizons of agents are determined by a factor τ, so that the average horizon is approximately two trading days τ_d of 100 trading sessions. I assume that the fundamental asset value is a white noise with volatility σ_ξ. During active trading, the price reverts to the fundamental value in 20 days. The fundamental price, order imbalance and noise impact coefficients are drawn from three independent exponential distributions for the entire population of traders: $\nu_1^i \sim \exp(1/\sigma_{\nu_1})$, $\nu_2^i \sim \exp(1/\sigma_{\nu_2})$, $\eta^i \sim \exp(1/\sigma_\eta)$, $\forall i = \overline{1, N_A}$, and are fixed during the course of trading. I run the simulations for a large set of values σ_{ν_1} and σ_{ν_2} in order to study the influence of different ratios between fundamentalist, imbalance and noise components on

[3] This assumption is compatible with requirements of many exchanges. For instance, the minimum peak size of an iceberg order on Euronext is 10 times the unit of trading.

[4] I define the concrete iceberg order submission scheme, while the mechanism of iceberg order execution and peak release are duly explained in Chapter 3.

[5] It is not feasible to draw a rigorous comparison between the degree of transparency of *Specification 3* and *Specification 4*: in the opaque market a potentially larger fraction of market volumes is displayed, however, the information about the orders that cluster around the best quotes is usually of higher importance since most of the trading happens there.

Table 4.2: Key input parameters of the numerical simulations.

Parameter	Value
number of agents	$N_A = 5000$
number of stocks	$N_S = 50$
initial fundamental price	$p_0^f = £300$
risk aversion	$\varphi = 0.1$
time horizon	$\tau = 200$
reversion horizon	$\tau^f = 20\tau_d$
day length	$\tau_d = 100$
noise volatility	$\sigma_\xi = 0.001$
fundamentalist impact	$\sigma_{\nu_1} = 0.1$
imbalance impact	$\sigma_{\nu_2} = 0.0037$
noise impact	$\sigma_\eta = 0.01$
tick size	$\Delta = £0.005$

price, order flows and the book, and establish that the results are robust within a fair interval from baseline values.

Each simulation run consists of 200,000 rounds. Since every market simulation commences with an empty limit order book, I discard the first 100,000 rounds and analyse the data from the remaining 100,000 rounds, which comprises 1,000 trading days. The profile plots and descriptive statistics of key time series for one simulation run of each specification are presented in Appendix F. In order to gauge how transparency regimes affect the collective behaviour of heterogeneous traders and what emergent market properties they generate, I replicate each simulation 100 times and compute average estimates of the key market statistics. In the subsequent sections I discuss the results of my numerical experiments in relation to empirical facts from double auction markets and to the principal research questions.

4.3 Stylised Facts

In this section I examine the ability of the market with heterogeneous trading rules to reproduce a number of stylised facts that are frequently detected in financial markets. In particular, I evaluate how the degree of market transparency alters the likelihood of abnormal price fluctuations, path dependence in order submissions, and other memory properties.

4.3.1 Abnormal Returns

The mean return is effectively zero under all specifications, whereas the diapason of returns expands significantly once traders have the opportunity to account for a limit order book depth to form their expectations of the future stock price (Table 4.3). As in the model of Chiarella et al. (2009) with noise, fundamentalist and chartist components, the reason for large price oscillations in a partially and fully transparent cases is the antagonistic effect of fundamentalist and imbalance components that override traders' expectations once a big price change occurs. However, in contrast with a quasi-transparent market which leads to the greatest amplitude of returns, the opaque microstructure helps to tame large price swings.

Table 4.3: Summary statistics of returns and order flow under four transparency regimes. The results represent averages over 100 simulation runs.

	Dark	*Transparent*	*Quasi-Transparent*	*Opaque*
Returns				
min	-0.0215	-0.0541	-0.0528	-0.0274
max	0.0246	0.0500	0.0559	0.0289
mean	0.0000	0.0000	0.0000	0.0000
s.d.	0.0014	0.0024	0.0024	0.0013
skewness	0.2320	-0.1527	0.1850	0.0842
kurtosis	18.8458	33.7944	34.9786	40.4401
Order Flow				
no. of orders	96,966.1	96,982.5	97,005.3	72,563.7
average order size	24.30	24.04	24.11	29.57
average BM	29.20	28.10	28.10	37.51
average BL	22.18	23.18	22.95	25.74
average SM	32.05	30.41	30.67	39.54
average SL	19.38	19.66	19.61	23.64

Though on average it is impossible to profit from trading in any of the considered markets, the opaque market offers the highest probability of receiving abnormal returns, as indicated by the kurtosis. Notably, only the transparent market embraces two empirical facts in the financial markets: asymmetric chances of receiving positive and negative returns and non-trivial probability of abnormal returns. This is a convenient result since a similar setup but with chartist impact in place of order imbalance impact, the model of Chiarella et al. (2009) could not replicate this empirical stylised fact of higher likelihood of negative returns.

4.3.2 Order Placement Patterns

In the simulations I distinguish between six classes of orders: a buy market order (BM), or an intention to buy the asset at a price higher or equal to the current best ask; a sell market order (SM), or an intention to sell the asset at a price lower or equal to the current best bid; buy limit orders placed inside the current bid-ask spread (aBL) and below the current best bid (pBL), and sell limit orders placed inside the current bid-ask spread (aSL) and above the best ask (pSL).

The general observation about the order placement is that the availability of information about book depth has no discernible effect on order flow in a transparent or a quasi-transparent market in that neither the cumulative number of submitted orders, nor the average order size change substantially. As indicated in Table 4.3, under these two transparency regimes the average size of aggressive orders diminishes, albeit insignificantly, and is accompanied by a marginal increase in the average size of passive orders. In an opaque market, the intensity of the order flow abates, though it is compensated by an increase in the average size of submitted orders.

First, I examine the order submission patterns to verify whether traders respond adequately to the observable state of the limit order book. More precisely, I check how order placements change depending on the market conditions against empirical inference that was consolidated in the work of Ranaldo (2004). Using order and transaction data from the Swiss Stock Exchange, which is a pure order-driven electronic stock market with no market maker, Ranaldo (2004) estimates a probit model and uncovers several determinants of order aggressiveness framed as the following properties:

P1: The wider the spread, the weaker the order aggressiveness.

P2: The higher the volatility, the stronger the order aggressiveness.

P3a: The thicker the book on the buy (sell) side, the stronger the order aggressiveness of the incoming buyer (seller).

P3b: The thicker the book on the sell (buy) side, the weaker the order aggressiveness of the incoming buyer (seller).

According to analytical results derived for Proposition 4.2.2, these properties hold on individual level for every trader. In order to verify the capability of the artificial market to reproduce these properties on aggregate, I compute the frequencies of all six order types submissions given the value of the corresponding market parameter. For instance, for the bid-ask spread variable I first count the frequencies of all order types submitted when the current spread is tight, then order frequencies when the spread at the time of submission is wide. Small and large sizes of the spread are defined against the average spread. In each simulation run I calculate the probability of submitting a concrete order when the parameter, e.g. the spread, is high minus the probability of the same order submission when the parameter is low.

As reported in Table 4.4, I detect the correlation between order aggressiveness and the spread size (P1) across all markets independently of the transparency degree: the spread widens, traders rely more on limit orders (the probability of orders inside the spread increases, and outside the spread decreases) and less on market orders. According to Ranaldo (2004), the volatility of returns and order aggressiveness move in the same direction. Also, I find that P2 holds only for sellers of the asset in all four cases. However, in contrast with the empirical evidence, higher volatility

Table 4.4: The changes in the probabilities of order types (in %) conditional on the market characteristics given an increase in the corresponding market parameter. The results represent averages over 100 simulation runs with the standard deviations provided in parentheses.

Dark market

	BM	aBL	pBL	SM	aSL	pSL
Spread	-6.29	12.45	-6.21	-5.08	12.38	-7.24
	(0.67)	(1.21)	(0.87)	(0.51)	(1.57)	(1.36)
Volatility	-3.80	0.63	3.06	1.47	1.47	-2.83
	(1.18)	(1.13)	(2.07)	(1.73)	(0.74)	(1.28)
Buy depth	1.12	-1.27	0.23	0.32	-1.47	1.08
	(0.73)	(0.81)	(0.56)	(0.49)	(0.95)	(0.85)
Sell depth	2.28	-1.70	-0.54	-0.69	-2.03	2.68
	(1.07)	(0.76)	(0.62)	(0.56)	(0.61)	(0.97)

Transparent market

	BM	aBL	pBL	SM	aSL	pSL
Spread	-4.96	9.72	-4.65	-4.31	9.84	-5.65
	(1.02)	(0.98)	(1.27)	(0.96)	(1.28)	(1.43)
Volatility	-3.80	0.43	3.51	1.07	1.35	-2.57
	(1.37)	(0.58)	(1.48)	(0.83)	(0.60)	(1.32)
Buy depth	4.77	-0.65	-3.77	-4.57	-1.74	5.96
	(2.04)	(0.83)	(1.48)	(1.88)	(1.23)	(2.40)
Sell depth	-4.30	-1.76	5.97	4.65	-0.75	-3.82
	(2.29)	(1.11)	(1.79)	(1.51)	(0.97)	(1.92)

Quasi-Transparent market

	BM	aBL	pBL	SM	aSL	pSL
Spread	-5.54	9.92	-4.40	-4.15	10.37	-6.22
	(1.07)	(0.98)	(1.22)	(0.99)	(1.28)	(1.65)
Volatility	-4.13	0.31	3.81	1.28	1.58	-2.85
	(1.27)	(0.77)	(1.43)	(1.07)	(0.62)	(1.46)
Buy depth	4.55	-0.40	-3.81	-4.07	-1.43	5.16
	(1.71)	(0.79)	(1.52)	(1.57)	(1.18)	(1.78)
Sell depth	-4.36	-2.04	6.36	4.53	-1.24	-3.25
	(1.92)	(1.23)	(1.72)	(1.34)	(1.04)	(1.36)

Opaque market

	BM	aBL	pBL	SM	aSL	pSL
Spread	-2.31	8.35	-6.25	-1.63	8.21	-6.38
	(0.95)	(1.06)	(1.40)	(0.65)	(1.28)	(1.28)
Volatility	-2.53	1.18	1.38	0.39	1.51	-1.93
	(0.84)	(0.43)	(0.91)	(0.71)	(0.40)	(0.81)
Buy depth	-11.98	2.63	11.35	-17.29	2.11	13.18
	(1.54)	(1.29)	(6.92)	(6.94)	(1.19)	(2.60)
Sell depth	-13.61	1.72	14.03	-12.96	2.00	8.82
	(4.51)	(0.86)	(3.55)	(3.50)	(0.88)	(3.51)

encourages buyers to fill the book. The asymmetry between the behaviour of patient buyers and sellers, who seek to close their positions quickly, is caused by the CARA preferences.[6] Regarding P3a and P3b, evidently, traders' submissions are affected by market depth only when it is disclosed to them to a certain extent. Once the information about the market depth becomes public and every agent can evaluate the demand and supply for the stock – precisely in a transparent market and with a bias in a quasi-transparent and opaque markets, – then traders start placing orders further away from the mid-point price and the shape of the book resembles a real market.[7] In this sense, accounting for market depth provokes a certain feedback effect. In transparent and quasi-transparent cases, an incoming buyer has a higher probability to use a market order and an incoming seller has an increased chance of trading via a passive limit order as the demand for asset increases. Concerning the aggressiveness of the order flow, the last two rows in Table 4.4 read that in an opaque market the more liquidity traders observe the higher is their propensity to provide liquidity by placing more passive orders.

Another aspect that characterises market quality is certain regularities or path dependences in order submissions. I examine the order flow pattern using an approach similar to Biais et al. (1995), who uncovered in their empirical study of Paris Bourse data a number of important properties including the so-called *diagonal effect*, whereabout orders of the same type tend to follow each other, so that the diagonal elements of the conditional distribution matrix have the highest weights. Typical conditional distributions of orders in the four markets are given in Table 4.5. Each estimate in this table stands for the number of times that the order type in row i was submitted after the order type in column j throughout 1,000 trading days. Dividing frequencies in each row by the total number of orders of the corresponding type, I convert to probabilities and then calculate a percentage deviation of a conditional probability of a particular order type from the unconditional probability of this order type. Along each column, the most likely order sequence is printed in bold. The last column shows the unconditional probabilities of six order types. Although the diagonal effect may not be a universal property of real markets, it is worth verifying its relation to the degree of quote transparency. The described evaluation method proves convenient for this purpose.

Overall, the diagonal effect is dissipated in my artificial marketplaces, unlike Biais et al. (1995),

[6]Higher volatility does not affect the expectations of the trader directly, but alters the region of admissible prices defined by p_m and p^*. According to Proposition 4.2.2, the subinterval of buy prices becomes shorter than the subinterval of sell prices proportional to the total interval length $p_M - p_m$ when the spot volatility is high. Consequently, it is more likely that the current market price of the security lies in the subinterval of selling prices (above p^*), and if the trader intends to purchase the asset, he submits a passive buy limit order.

[7]In this model, as Chiarella et al. (2009) duly explain, a market with pure noise traders is characterised by a substantial asymmetry of a limit order book towards the buy side, which is a consequence of CARA preferences. This imbalance partly evens when traders become aware of the fundamental asset value and then orders cluster around the best quotes. In my version of the model the positive skewness in returns, however, is not removed, as was pointed out for a dark market.

Table 4.5: Percentage deviations of order type probabilities conditional on the previous order type from their unconditional probabilities in the last column. The results represent averages over 100 simulation runs.

Dark market

	BM	aBL	pBL	SM	aSL	pSL	Unconditional
BM	**28.0**	2.5	-8.5	-35.4	1.6	14.3	16.61
aBL	24.4	-1.9	-10.4	3.7	-4.8	-7.3	11.22
pBL	-34.6	2.4	**11.9**	22.2	**4.2**	-7.7	23.50
SM	-34.8	0.7	10.5	**26.3**	3.9	-8.3	17.09
aSL	6.1	-0.0	-4.8	19.4	-4.5	-13.1	11.06
pSL	25.2	**3.2**	-8.1	-36.8	3.2	**16.1**	20.53

Transparent market

	BM	aBL	pBL	SM	aSL	pSL	Unconditional
BM	**35.0**	**3.5**	-12.3	-39.0	-2.3	**14.5**	15.89
aBL	33.6	-6.2	-11.9	2.1	-9.0	-4.9	9.17
pBL	-37.6	1.6	**14.1**	24.6	6.6	-9.0	25.74
SM	-42.6	2.5	13.3	**35.1**	**9.3**	-13.1	15.51
aSL	6.3	-5.2	-3.6	26.7	-6.7	-12.8	9.18
pSL	23.0	2.2	-7.4	-32.3	-0.6	12.7	24.51

Quasi-Transparent market

	BM	aBL	pBL	SM	aSL	pSL	Unconditional
BM	**35.9**	**5.2**	-13.6	-40.7	-0.8	**16.7**	16.06
aBL	30.8	-5.5	-11.9	3.0	-7.3	-4.9	9.40
pBL	-36.4	0.3	**14.2**	24.4	5.0	-9.7	25.80
SM	-41.4	0.7	13.9	**33.8**	**6.7**	-13.3	16.09
aSL	5.6	-5.5	-3.3	26.4	-5.4	-14.1	9.46
pSL	26.0	4.1	-9.2	-35.8	0.3	15.3	23.19

Opaque market

	BM	aBL	pBL	SM	aSL	pSL	Unconditional
BM	**31.8**	**3.7**	-11.0	-34.2	1.8	**13.2**	16.79
aBL	27.5	-6.4	-9.5	2.1	-9.2	-5.4	8.27
pBL	-32.9	1.4	**11.7**	23.6	3.7	-8.9	26.07
SM	-35.8	2.5	11.2	**29.3**	**5.1**	-11.4	17.51
aSL	10.5	-5.9	-4.9	17.2	-8.3	-10.0	8.08
pSL	23.8	2.5	-7.9	-30.1	1.7	12.8	23.27

where it emerges so evidently.[8] This said, I observe significant path dependence in market orders equally for buyers and sellers, which is further facilitated by increased market transparency. Series of passive limit orders are slightly more rare, though such orders are still more likely to arrive in batches rather than independently. Chordia et al. (2002) document that positive returns tend to

[8] I translate the order classification of Biais et al. (1995) into my order classification in Appendix E.

be continued, whereas negative returns most frequently provoke market reversals. This evidence corresponds to a weakening diagonal effect in the domain of sellers: while buys – especially aggressive market orders – tend to arrive one after the other and push the transaction price up, sell orders appear to be less likely to cluster in this manner. The differences in estimated conditional order distributions in Table 4.5 suggest that quote transparency generates stronger regularities in order submissions, although the magnitude of discrepancies across four market specifications is not dramatic.

The justification of the diagonal effect, according to Biais et al. (1995), is at least threefold: (1) strategic order splitting, (2) imitation, or (3) similar expectations. Any conscious imitation or strategic behaviour is redundant in the present model, hence the sole reason for traders to copy each other is some kind of herding mechanism, facilitated by matching expectations. All traders possess the same market depth information and have identical fundamental asset valuation. On the other hand, an order placed by the previous trader always affects the expectations of the one arriving after him by changing the depth and possibly the distance to the fundamental price. In the case with visible order imbalance the heterogeneity in trading rules amplifies, resulting in larger discrepancies in the future price expectations and distinct order placement across agents. Therefore, in the absence of intentional imitation between market participants this framework captures these similarities in traders' order placements. Moreover, this artificial market reproduces fairly symmetric conditional distribution of order types alike Paris Bourse data of Biais et al. (1995).

In terms of unconditional probabilities, as shown in the right column in Table 4.5, most of the traders are liquidity providers in the environments with and without market depth information. Notice, however, that the order flow is more symmetric in the former case. Concerning the unconditional probabilities of six order types in an opaque market, I compare the distribution to the empirical data from Paris Bourse in Biais et al. (1995) and the Australian Stock Exchange in Hall and Hautsch (2006), both of which permit the use of hidden orders for large trades.[9] In the data from Paris Bourse, the sellers, and most impatient ones, outnumber the buyers. On the contrary, out of five stocks from the Australian Stock Exchange analysed by Hall and Hautsch (2006) buyers dominate in two markets compared to fairly symmetric order flows in the other three. In terms of order flow pattern and overall order flow aggressiveness the simulated opaque market bears the closest resemblance to the market for News Corporation shares: the proportions for buy and sell order are symmetric with a large fraction of passive limit orders. In their recent study of 100 stocks listed on INET in 2004, Hasbrouck and Saar (2009) find that not only limit orders compose the bulkiest part of the order flow, they also tend to have larger size than market orders. However, in many instances limit orders are cancelled within seconds after submission, and these fleeting, or flash orders usually exceed in size the limit orders that remain patiently in the book awaiting execution. This empirical

[9]See Table 4.12 and Table 4.13 in Appendix E.

evidence overturns the traditional treatment of limit orders as passive liquidity provision strategies. Instead, such application of big limit orders embodies aggressive liquidity-seeking behaviour that depletes the order book and harms market liquidity. Therefore, given the bias of fleeting orders, it is hard to retrieve the strength or liquidity streams from empirical data.

Further, the estimates in Table 4.5 indicate that the probability of market orders in a quasi-transparent market increases accompanied by a small increase in the probability of orders inside the spread, that is, aggressive limit buys (aBL) and sells (aSL). In the study of Yamamoto (2011), where traders react to the order imbalance by means of probabilistic switching mechanism, once the information about order volumes recorded in the book becomes limited, the use of market order slightly escalates in comparison to a perfect transparency case. Similarly, in my framework, once traders are constrained to observe only the best quotes, market orders accrue a slightly higher share in the order flow relative to perfect transparency. This partially explains a little revival in the trading activity: in this market a higher number of transactions leads to a faster turnaround of assets. Comparison between a transparent market and an opaque market confirms the empirical findings of Frey and Sandås (2009) that the use of market orders intensifies as a reaction to increased opacity.

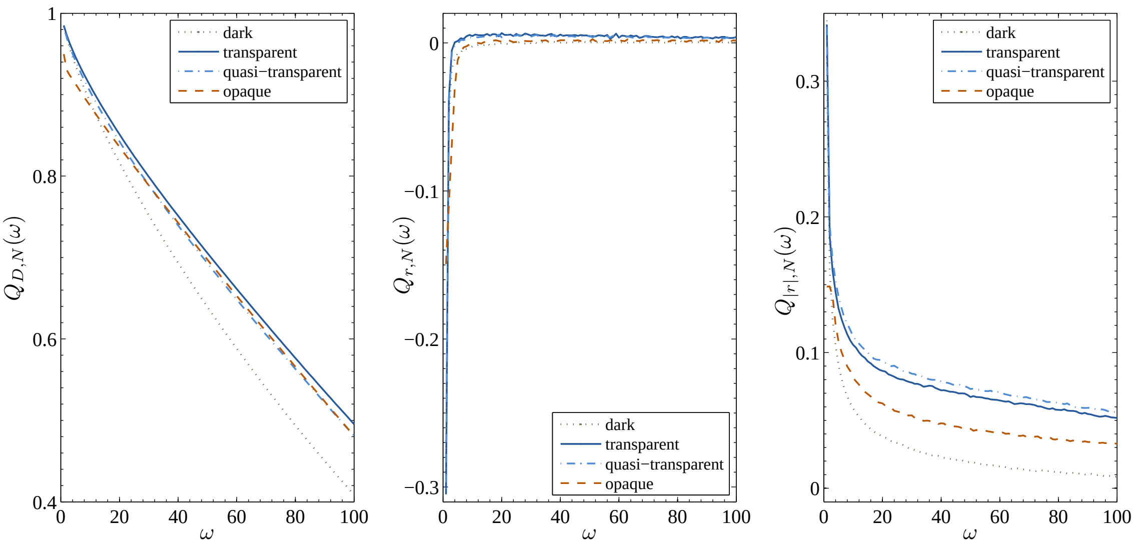
4.3.3 Memory Properties

Memory in the financial time series stipulates predictability of future market dynamics, thereby it creates a relevant argument for the topic of market efficiency. I examine the memory effects in three variables which have an immediate impact on agents' order placement: returns, absolute returns and order imbalance. Figure 4.1 depicts the autocorrelation functions for 1 to 100 lags in returns and imbalance for the four model specifications. The total order imbalance variable pertains a strong memory, as Figure 4.1(a) demonstrates. The memory decays faster for the dark market and is most persistent for the transparent market. Reducing market transparency helps to diminish the long term path dependence in the order imbalance, whereas permitting the use of iceberg orders decreases the short term memory as well. I calculate Ljung-Box statistic to test the significance of the autocorrelation effect in the series:

$$Q_N(\omega) = N(N+2) \sum_{\tau_\omega=1}^{\omega} \frac{\rho^2_{\tau_\omega,N}}{N-k} \sim \chi^2_{1-\alpha,\omega}, \tag{4.4}$$

where $\rho_{\tau_\omega,N}$ is the sample autocorrelation coefficient for lag τ_ω and N is the sample size. Based on Ljung-Box test I find that serial autocorrelations in absolute returns and in order imbalance remain statistically different from zero for all 100 included lags and further. The simulation outcomes of Yamamoto (2011) also confirm substantial persistence in order imbalance both for transparent and quasi-transparent market setups. Moreover, Lillo and Farmer (2004) detect long memory in the order flow, particularly in the signs of submitted orders, for a range of stocks listed on the London

Stock Exchange and sampling years. This result is connected to my artificial markets in two ways. First, it justifies the autocorrelations in Figure 4.1(a). Take, for example, successive arrival of sell orders: limit orders add depth to the sell side and increase the order book imbalance, as defined in equation (4.1); market orders absorb the volume on the buy side which again has a positive impact on D_t. Second, the order signs memoryness echoes the discussion of the diagonal effect in submitted order sequences. In their review of the statistical evidence on long memory in order flow Mike and Farmer (2008) argue that long memory property in order signs has a profound impact on price dynamics and induces a power law distribution with high probabilities of extreme returns. They explain it by order splitting and other dynamically optimised strategies, whereas in my setup it is rather the unconscious herding caused by matching price expectations that is responsible for the correlation in buy and sell volumes.



(a) (b) (c)

Figure 4.1: Autocorrelation functions of (a) order imbalance, (b) returns, and (c) absolute returns series estimated as averages over 100 simulation runs for each of the four specifications.

On the contrary, Figure 4.1(b) shows that there is no persistent correlation in returns apart from the first few lags, which implies informational efficiency regardless of the degree of market transparency. The negative autocorrelation in returns for small lags is caused by the bid-ask bounce, generated by the impatient traders who demand liquidity. This effect is the smallest for the market with no depth information and the market with a hidden layer, which is explained by the thicker limit order book in both of these market setups. The correlations are statistically significant up to up to lag 67, 75 and 72 in the dark, transparent and quasi-transparent markets respectively, and only for the first 27 lags in the opaque market at a 5% confidence threshold. Therefore, the memory in returns dissipates most quickly in the opaque market where hidden depth is created by iceberg orders. Regarding the autocorrelation in absolute returns that is usually interpreted as a proxy of volatility, Figure 4.1(c) shows a positive dependence in the absolute returns even for distant lags. Overall, the higher the transparency, the stronger the memory in absolute returns. However, the impact of restricting the info only to the five best quotes is somewhat ambiguous since it intensifies the long memory in absolute returns compared to the full depth visibility case. The combination of uncorrelated returns and significant positive dependence in absolute returns quantifies the volatility clustering.[10]

In addition to the autocorrelation analysis I apply a less conventional method to screen the long memory properties in asset returns and volatility. I calculate the modified rescaled range statistic that was proposed by Lo (1991) as an improvement over the original Mandelbrot (1972) test. This nonparametric test is more appropriate in the context of stark departures from the normal distribution in the data. The test also allows to discard conveniently short period memory and concentrate mainly on long term correlations. The test statistic is given by:

$$\tilde{Q}_n(\omega) = \frac{1}{\sqrt{V_n(\omega)}} \left[\max_{1 \le k \le n} \sum_{i_k=1}^{k} (x_{i_k} - \bar{x}_n) - \min_{1 \le k \le n} \sum_{i_k=1}^{k} (x_{i_k} - \bar{x}_n) \right], \tag{4.5}$$

where $n < N$ is the subsample size drawn from N-observations sample, $\bar{x}_n$ is the subsample mean, V_n is the subsample variance, and the covariance parameter $V_n(\omega)$ is calculated using subsample autocorrelation coefficients $\rho_{\tau_\omega,n}$ for each lag τ_ω as $V_n(\omega) = V_n + 2\sum_{\tau_\omega=1}^{\omega} \frac{\omega+1-\tau_\omega}{\omega+1}\, \rho_{\tau_\omega,n}$. The estimate of statistic $\tilde{Q}_n(\omega)$ is adjusted to the subsample size by calculating the coefficient $\beta_n(\omega)$ that determines the result of the modified rescaled range test:

$$\beta_n(\omega) = \frac{\ln \tilde{Q}_n(\omega)}{\ln n}. \tag{4.6}$$

In case $\beta_n(\omega) < 0.5$, there is a mean reversion in the data series. If $\beta_n(\omega) > 0.5$ there is a positive correlation in the data and a long memory property.

Applying the modified rescaled range test, I monitor the evolution of memoryness in absolute returns for various time horizons. The strong dependence in absolute returns depicted in Figure 4.2(a) supports the volatility clustering phenomena from real financial markets: there is a slow decay in the serial correlation of the volatility of asset returns. As more lags ω are omitted, the coefficient $\beta_n(\omega)$ decreases and slowly approaches the benchmark level 0.5, which indicates that memory of distant past absolute returns taken in isolation fades away, hence the spot volatility of returns is independent in the long term. In other words, periods of high volatility in returns are followed by

[10] In terms of graphical analysis, (Figure 4.6(b), Figure 4.7(b) and Figure 4.8(b)) depict clustering in returns series under three transparency regimes, compared to the more monotonous variability of returns in the dark market in Figure 4.5(b)).

periods of low volatility. The clustering is very poor in the dark market, which is indicated by low values of $\beta_{|r|,n}(\omega)$ even for small lags, in contrast with transparency regimes where all or part of the book volume is revealed to traders.

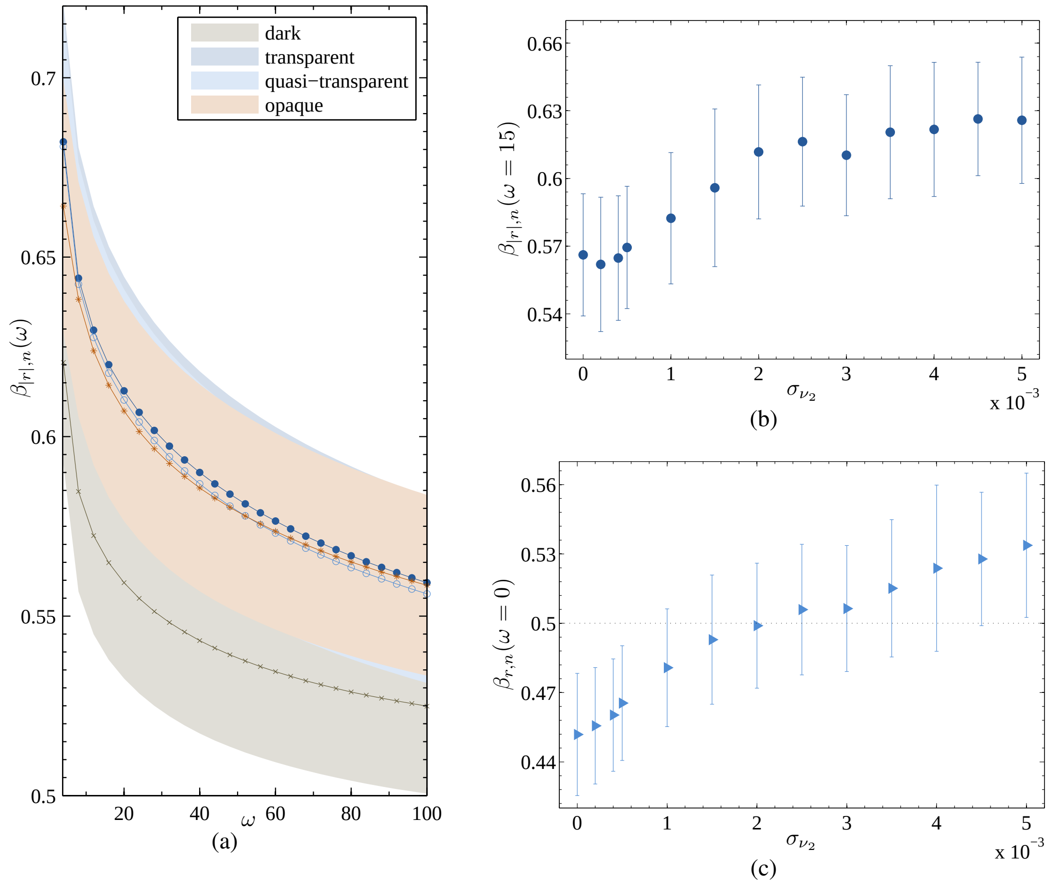


Figure 4.2: Modified rescaled range test coefficient of (a) absolute returns for various lags (error bars computed over 100 simulations), (b) returns in a transparent market with all lags included, and (c) absolute returns in a transparent market with the optimal number of discarded lags. The subsample size is 5,000 time steps.

The choice of the truncation lag ω for the long memory examination requires some consideration. On the one hand, the parameter ω cannot be very small to avoid the prevalence of short term correlations which may be substantial. On the other hand, several Monte Carlo studies, including Lo and MacKinley (1989) and Andrews (1991), have shown that inappropriately large ω relative to the subsample size can distort the convergence properties of the estimator $\beta_n(\omega)$ in a finite sample and deem the test inadequate. I apply the criterion suggested by Lo (1991) to select the optimal number of lags:[11]

$$\omega^{opt} = \left[\left(\frac{3n}{2} \right)^{1/3} \left(\frac{2\rho_{1,n}}{1-\rho_{1,n}^2} \right)^{2/3} \right], \tag{4.7}$$

where n is the subsample size and $\rho_{1,n}$ is the first lag autocorrelation estimate. Taking this rule as a guideline, I investigate how the intensity of order imbalance impact on traders' price expectations affect the memoryness of returns and absolute returns interpreted as latent volatility. The results across the three market specifications with partial or full quote visibility are reasonably similar, thus I present just the estimates for the transparent case. Figure 4.2(c) depicts the standard rescaled range coefficient for asset returns with all lags included as a function of imbalance impact σ_{ν_2}. When order imbalance impact is meagre, the returns clearly exhibit a mean reversion pattern. However, some positive correlation emerges as the standard deviation of imbalance weight of imbalance grows above 0.005. The short term dependencies in absolute returns converge to zero at the horizon ω^{opt} = 15, according to (4.7).[12] The graph of modified rescaled range coefficient in Figure 4.2(b) shows that even for low values of imbalance component there is a long memory effect, albeit moderate. Higher weight of imbalance in traders' price predictions, at the same time, translates into more persistent long range dependencies in the absolute returns which match empirical findings (Lo, 1991). The inclusion of a sizable imbalance weight is therefore an important aspect of this model, given that it allows to replicate more accurately real market dynamics. In addition, this analysis highlights that the combination of baseline parameters for my artificial market implies memoryless returns (the confidence interval of $\beta_{r,5000}(\omega = 0)$ overlaps with 0.5 in this instance), and strong long term correlations in absolute returns ($\beta_{|r|,5000}(\omega = 15) \approx 0.62$). The model is also robust with respect to reasonable changes in σ_{ν_2}, as follows from Figure 4.2(c) and Figure 4.2(b).

4.4 Markets Quality Analysis

In this section I explore the consequences of reduced pre-trade transparency regimes on the market quality characteristics and discuss the main findings of my simulation analysis.

4.4.1 Price Discovery and Transaction Costs

The comparison of transaction price trajectories generated independently in the four markets reveals that the availability of market depth information decreases the mean transaction price. As the preliminary order flow analysis has demonstrated, full quote transparency mitigates traders' aggressiveness, limit orders tend to be smaller and more scattered in the book. The estimates in Table 4.6 reveal that the average transaction price deviates substantially from the fundamental value in

[11] The underlying assumption of this rule is that the data conform to an autoregressive process of order one.
[12] The autocorrelation coefficient $\rho_{1,5000}$ is approximately 0.3.

the transparent market because agents that demand immediacy in this market trade through several limit orders straightaway, skipping gaps in the book. Hence, patient traders, that submit their orders inside the book and make no revision, suffer from the winner's curse, for instance: once a buyer reaches the front of the bidding queue, he gets a quick fill by an impatient seller, but the valuation of the asset has been changed already, hence the buyer receive a bad price. The restriction of displayed depth to the five best quotes in a quasi-transparent market only exacerbates these departures.[13] This rise in the volatility of prices as well as returns afflicts risk-averse investors and leads to a decline in the market activity, measured by the total number of executed trades and the cumulative trading volume over the whole 1,000-days period. In contrast, the opaque environment assists price discovery and the average transaction price remains close to the fundamental stock value. According to the mechanism of traders' expectations formulated in Proposition 4.2.2, sellers are motivated to hide their order size to alleviate the selling pressure and thus draw more buyers to the market, and vice versa for the buyers. This mechanism translates into higher efficiency in the opaque market relative to both transparent and quasi-transparent markets.

Table 4.6: Statistics of the bid-ask spread, trading volumes and prices under four transparency regimes. The results represent averages over 100 simulation runs.

	Dark	*Transparent*	*Quasi-Transparent*	*Opaque*
Spread				
mean	0.40	0.68	0.67	0.46
s.d.	0.38	0.68	0.69	0.42
Transactions				
no. of trades	73,746.4	67,948.6	69,297.9	74,453.6
trading volume	1,004,221.9	889,381.9	919,896.9	957,324.7
average trade size	13.62	13.15	13.28	12.86
average trade price	189.15	186.26	185.30	189.39
average fundamental price	191.45	191.45	191.45	191.45

Yet, a more dramatic consequence of enhanced quote transparency is the widening of the bid-ask spread by almost 70% compared to the dark market. Furthermore, my experiments show that exogenous restriction of transparency only accentuates the negative effect on transaction costs proxied by the spread. The contrast in evidence for the transparent and quasi-transparent markets is coherent with the adverse selection argument. On the contrary, in the opaque environment transaction costs, estimated by the bid-ask spread, decrease substantially and stimulate trading activity, as can be deduced from an increase in cumulative transactions. This result is in line with the experimental

[13]Figure 4.6(a) and Figure 4.7(a) illustrate this for the time series of the transaction price superimposed on the fundamental price.

findings of Bloomfield and O'Hara (1999) and the empirical study of Madhavan et al. (2005).[14] The authors claim that full transparency augments the problem of adverse selection and thereby widens the spreads. From this viewpoint, lower levels of transparency introduced by iceberg order trading improve market quality.

4.4.2 Liquidity Indicators

In the comparative examination of transparency regimes, I apply several methodologies to evaluate the key aspects of market liquidity, such as depth, breadth and resilience of the limit order book, and immediacy.

Market Depth

In terms of visible market depth, any increase of book transparency has an advantageous impact. I observe that full disclosure of market depth as well as endogenous disclosure regulation in the opaque market have a positive effect on the balance between total buy and sell volumes in the book. The effects of reduced pre-trade transparency on the book depth and trading volumes coincide with the effects documented by Aitken et al. (2001) and Anand and Weaver (2004) in their recent empirical research of the Australian and Toronto Stock Exchanges respectively. The visible order book imbalance in a quasi-transparent market is almost zero on average, whereas the overall imbalance is more inclined towards buyers and reverts to the level of the dark market. Thereby, restricting the visibility of the book unilaterally generates more uncertainty, the asset value depreciates and more buyers queue in the book.

Most of the trading in the markets occurs near the best quotes, however, even full pre-trade transparency does not remove the imbalance between the buy and the sell depth. With only few quotes displayed, the risk of non-execution is mitigated by the agents' over-reaction to the observed imbalance and drastic price swings become less rare. The visible volume in the opaque book is well balanced between the buy and sell sides as in a quasi-transparent setting. The displayed volume accounts for approximately 42% symmetrically for buyers and sellers even though hidden volume is more likely to be created on the buy side of the limit order book than on the sell side or inside the spread. The fluctuations in the visible order imbalance are minor relative to the outcomes of the rest model specifications. The balance between aggregate demand and supply improves: an average gap between buy and sell volumes shrinks to 7% of the total market depth in the opaque market, while in a quasi-transparent market the cleavage accounts for 21%.

[14] My experiments do not control for the direction in transparency regime changes, which proves essential in the empirical implementation of transparency rules. Anand and Weaver (2004), for instance, show that a ban and a subsequent reintroduction of iceberg orders cause different market reactions. With this caveat in mind, I only compare the correlations between the direction of the transparency regime change and the ensuing movements in market indicators.

Table 4.7: Statistics of market depth and imbalance under four transparency regimes. The results represent averages over 100 simulation runs.

	Dark	*Transparent*	*Quasi-Transparent*	*Opaque*
Visible Imbalance				
mean	0.00	0.2537	0.0700	0.0923
s.d.	0.00	0.6787	0.7221	0.4816
Total Imbalance				
mean	0.3619	0.2537	0.3568	0.1514
s.d.	0.7346	0.6787	0.7291	0.6424
Visible Book Depth				
average buy depth	0.00	855.52	80.48	339.43
average sell depth	0.00	555.80	83.68	290.08
average book depth	0.00	1,411.32	164.16	629.51
Total Book Depth				
average buy depth	615.93	855.52	811.16	812.11
average sell depth	404.31	555.80	523.55	699.42
average book depth	1,020.24	1,411.32	1,334.71	1,511.52

Conditional Order Aggressiveness

Order flow is an undoubtedly rich source of information regarding liquidity motives of market participants. In order to dissect these liquidity patterns I assign order sequences into the following categories:

- undercutting/boom of liquidity provision (UB),
- resilient liquidity (RS),
- transient liquidity (TR), and
- depletion of liquidity (DL).

The grouping of 36 possible order placement events into these four categories is visually represented in Table 4.8. Property UB is the aggregate probability of those order sequences when there is an intensive stream of limit orders added to both sides of the book, or there is undercutting of passive limit orders by aggressive limit orders suggesting a tight competition for liquidity provision. The second property RS unites the events whereupon a passive order succeeds an aggressive order: when the latter sweeps liquidity at the best quote or quotes, then a new order arrives to replenish consumed liquidity. Transient liquidity TR is a mirror situation to resilient liquidity: if a limit order improves market depth, the ensuing trader fills orders on the same side of the market. Lastly, property DL indicates a chain of aggressive trades.

For the quantitative assessment of the defined patterns I sum unconditional probabilities of those events that fall within the corresponding group as highlighted in Table 4.8. As follows from

Table 4.8: The diagram of liquidity provision patterns based on conditional order distributions.

	BM	aBL	pBL	SM	aSL	pSL
BM	DL	–		DL	TR	
aBL pBL	–	UB		RS	UB	
SM	DL	TR		DL	–	
aSL pSL	RS	UB		–	UB	

Table 4.9: The estimates of liquidity provision patterns based on conditional order distributions. The results represent averages over 100 simulation runs.

	Dark	*Transparent*	*Quasi-Transparent*	*Opaque*
UB	43.85	46.96	45.90	43.02
RS	13.12	12.76	13.02	13.46
TR	12.13	11.87	12.10	12.37
DL	10.90	9.58	10.02	11.51

Table 4.9, there is no marked difference in the distribution of the four patterns across transparency specifications. Property UB is the most prominent in all market types, which is line with the fact that order flow is mostly composed of passive orders that fill a limit order book. Also, UB is higher in a dark and opaque markets: indeed, the rotation among the traders that who place orders inside the book indicates increased competition among traders to provide liquidity, as registered in cumulative trading volumes. Property TR is a little stronger in a market with partial information about the volumes, suggesting that liquidity is sporadic. However, with iceberg order release mechanism in hindsight, a large market order triggers multiple iceberg renewals which inflates TR figure, while the true transience of liquidity is actually weaker. The resilience property of the book mildly subsides as agents gain full information about quote depth and improves in an opaque market. There is a high demand for immediacy on both sides of the book and arriving agents trade with existing orders and widen the bid-ask spread, as indicated by property DL.

Liquidity Provision

Unlike quote-driven markets where a market maker is responsible for supplying liquidity, in pure order-driven markets traders themselves generate liquidity. In this contest the prevalence of passive orders that fill the book or, on the contrary, aggressive orders that consume market depth provides an insight into the market liquidity level. I define another liquidity provision indicator as a proportion of the volume available in the book to the total volume of incoming limit and market orders combined, hence the value of this indicator is bounded between zero and one. An abundant passive liquidity in the market with a weak stream of incoming market orders is associated with large liquidity provision ratio, signifying a substantial divergence in the sellers' and buyers' valuation of the asset. A liquidity provision ratio close to zero implies that there is a high demand for immediacy and insufficient liquidity in the market. A balanced market, therefore, is characterised by a liquidity provision in the region of 0.5, when there is an active trading with a fair market depth at any point in time.

Simulation outcomes in Table 4.10 reveal that the transparent market generates the highest liquidity provision coefficient, whereas the quasi-transparent market reconciles the demand and supply of liquidity. This observation reflects that availability of market depth information increases the heterogeneity between agents and thickens the limit order book, which, in turn, impedes the price discovery process despite the common observable fundamental stock price. Conversely, limited market depth disclosure accelerates price discovery by balancing passive agents and those who seek immediacy. In this respect the opaque market furnishes the most efficient rules.

Table 4.10: The summary of liquidity indicators under four transparency regimes. The results represent averages over 100 simulation runs with the standard deviations provided in parentheses.

	Dark	*Transparent*	*Quasi-Transparent*	*Opaque*
Liquidity provision	0.5729	0.6152	0.6052	0.5504
	(0.0146)	(0.0375)	(0.0299)	(0.0425)
Execution delay	0.0174	0.0120	0.0122	0.0008
	(0.0017)	(0.0011)	(0.0010)	(0.0002)
Kyle's lambda	0.0071	0.0114	0.0111	0.0019
	(0.0026)	(0.0035)	(0.0031)	(0.0009)

Order Execution Delay

Given that traders submit orders of various sizes, I measure next a volume-weighted average delay in limit order execution that is recorded from the moment of reaching the front of the queue to the full execution or expiry as a fraction of traders' horizons. Following this definition, the execution delay lies in an interval between zero and unity. In this example I choose model parameters such that most of the traders are day traders, with the longest possible execution time is slightly over two days. The present definition of delay matches the notion of aggregate liquidity factor introduced through execution delay earlier in Chapter 2. However, here, to account for the heterogeneity in traders time horizons I measure a delay relative to the individual time horizons. The comparison of the descriptive statistics (Table 4.10) of execution delays suggests that liquidity (inverse of the mean delay) increases with the arrival of iceberg order traders, while uncertainty (the variance) remains almost unchanged. Figure 4.3 plots the average distributions of the volume-weighted average delays for the four specifications in order. The opaque market followed by the quasi-transparent market with exogenously restricted transparency exhibit the most prominent skewness of execution delay towards zero. Therefore, patient traders benefit from imperfect transparency and, at the same time, they are better off than in the dark case. However, as was emphasised in the previous subsection, only the former setup of the two offers faster execution at no extra adverse selection cost. Furthermore, I apply a nonparametric test to establish formally whether execution times arise from the same distribution.[15] The results of the test confirm the conclusions from graphical analysis.

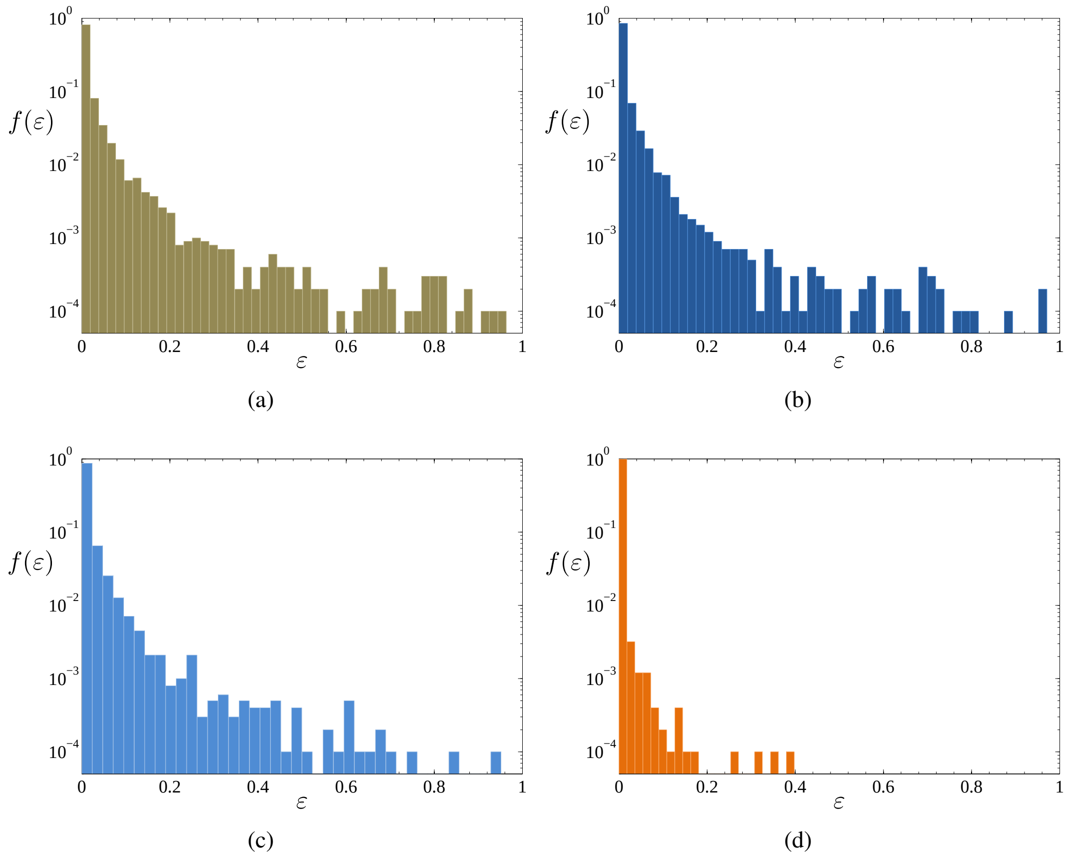


Figure 4.3: The distribution of execution delays in (a) a dark market, (b) a transparent market, (c) a quasi-transparent market, and (d) an opaque market. The market parameters are given in Table 4.2.

[15]The detailed test procedure is described in Appendix D.

Price Impact

Finally, I obtain the estimates of the liquidity indicator proposed in the paper by Kyle (1985), usually referred to in the literature as Kyle's lambda. This indicator essentially embraces several aspects of liquidity including market resilience and depth, and is associated with the price impact of aggressive trades. For each executed trade I record the signed order flow q_t that is assumed positive for the buyer-initiated trades and negative for seller-initiated trades, the change in the price Δp_t inflicted by the trade and the average asset return $\bar{r}_t$. Using the data from 100,000 trading rounds I estimate the following equation:

$$\Delta p_t = \lambda^K q_t + \mu \bar{r}_t + \zeta_t. \tag{4.8}$$

The coefficient $\hat{\lambda}^K$ is reciprocal to liquidity: the higher is $\hat{\lambda}^K$, the more drastic effect an aggressive order makes on the limit order book depth and the wider is the resulting spread.

The lowest estimate of Kyle's lambda is for the opaque market which is entirely consistent with the idea of exposure control via iceberg orders. Consider, for example, a large market sell that exhausts all visible volume at the best bid and then new peaks are released at this price providing additional fills to the impatient seller. The same mechanism is activated for the neighbouring buy quotes, and thereby slows down the seller walking down the book and alleviates the total price decrease created by his aggressive trade. The next best market in terms of price impact appears to be the dark market, according to results in Table 4.10. However, recall that unlike real order books a simulated dark market generates intensive order clustering around the best quotes with rare submissions further away. This is the reason why under this market specification the impact of trades is moderate – there are simply too few counterparties far from the current asset price. Hence, the value of $\hat{\lambda}^K$ in this case is not very indicative. Regarding the empirical findings on this topic, Anand and Weaver (2004) apply this comprehensive liquidity measure and find that neither a ban, nor a reintroduction of hidden orders provokes liquidity adjustments. Exogenously reduced transparency pertains a marginally smaller price impact than perfect transparency in my simulations. I conclude that intermediate levels of transparency in displayed quote depth, and particularly the opaque microstructure, stimulate liquidity measured by the Kyle's lambda.

Overall, on the basis of five different approaches the opaque market appears to be the most liquid trading environment.

4.5 Conclusion

This chapter provided an extensive analysis of the impact that various pre-trade market transparency rules produce on market performance. The evidence on benefits and adverse effects caused by limited market transparency is gained in the artificial double auction market simulations. Two distinct reduced transparency regimes were included in my analysis: a market with visible orders restricted to five best quotes on each side of the book – a quasi-transparent market, and a market with a permission to use iceberg orders type for large trades – an opaque market. I review here the main findings and prescriptions of my model.

The core implication of my agent-based model is that restricting the information about the market depth improves efficiency of order-driven markets, as indicated by the diminishing memory properties of returns and absolute returns alike. This applies both to the explicit restriction of the displayed quotes on behalf of the exchange and to the permission of iceberg orders. In the latter case, once traders are allowed to self-regulate the degree of transparency by managing their exposure with iceberg orders, the market becomes less predictable in short term as well as at longer time horizons. The transparency level imposed by certain exchanges whereupon only several best orders on each side of the limit order book are publicly displayed is controversial. This policy clearly has a positive impact on the speed of trading: the cumulative transacted volume increases while the non-execution risk of limit orders moderates. This, however, is an example of a winner's curse situation, since patient traders get quick execution for their orders but the price of the security moves away. Given this, traders are more prone to seek immediacy and hesitate to fill the book, which, in turn, becomes on average thinner than under full depth disclosure rule. Not surprisingly, the control over disclosed depth information impedes the price discovery process, reflected by the deviations of transaction price from the fundamental asset value.

Reduced market transparency also protects patient limit order traders from adverse selection to a certain degree, and, as a consequence, relieves the average transaction costs in the market measured by the bid-ask spread. The opaque microstructure has a profound impact on the spread and its volatility, while in a quasi-transparent market the book remains asymmetric with buyers dragging the price down. The ample evidence from empirical literature does not reach a consensus on the desirability of pre-trade transparency. As shown by the experiments of Bloomfield and O'Hara (1999), market opacity leads to wider bid-ask spreads, on the other hand, Flood et al. (1999) observe the reverse effect in the inter-dealer market. A recent comparative study of an introduction, ban and subsequent reinstallment of hidden orders on the Toronto Stock Exchange conducted by Anand and Weaver (2004) does not detect any discernible changes in the spread width. Moinas (2010) rationalises in the sequential trade model with high transparency level that limit order traders are reluctant to offer free options to other traders, thus market liquidity shrinks. The reaction to the increased risk of being picked off was discussed in earlier literature by Copeland and Galai (1983); Foucault (1999); Seppi (1997) among others.

It follows from the experiments reported in this chapter that markets with hidden layers ensure higher liquidity ratio, faster execution and smaller price impact of aggressive trades. Also, such markets are more active and carry out substantial trading volumes in line with Aitken et al. (2001);

Anand and Weaver (2004); Madhavan et al. (2005). Restricting quote transparency to several best quotes has negligible influence on market quality. Based on the numerical market simulations, Yamamoto (2011) concludes that the level of pre-trade transparency is not crucial in terms of market performance and long memory. It is coherent with the real market observations that suggest that most of the asset pricing information is encapsulated in the best bid and ask, and any information beyond the best quotes has marginal influence on the direction of trading. Moreover, the distribution of the recorded limit orders under restricted quote information remains rather sparse as in the case of perfect transparency and abundant aggregate market depth does not result in liquidity improvement since many of the orders are placed too far away from the quotes and expire unfilled. At the same time, the gaps in the book result in more dramatic price swings and higher volatility in returns as compared to the absolutely transparent market environment. Issuing the traders a permission to choose themselves the revealed order size allows to circumvent these problems. The empirical outcome closest to my opaque market simulations was achieved by Madhavan et al. (2005) who propose a theoretical framework and reconcile it with the Toronto Stock Exchange data.

In summary of this chapter, full quote transparency incurs substantial transaction costs for investors and dampens trading activity in an order-driven market, and exogenous restriction of displayed quote depth does not alter substantially the dynamics. On the other hand, the endogenous restriction of displayed quote depth via iceberg orders appears to improve market quality in multiple dimensions.

Appendix A Detailed Model Description

When called to trade at time t, agent i builds his expectation of the return on the asset that can be achieved within his time horizon τ^i according to the rule:

$$\hat{r}^i_{t,t+\tau^i} = \frac{1}{\nu^i_1 + \nu^i_2 + \eta^i}\left[\nu^i_1 \frac{\ln(p^f_t/p_t)}{\tau^f} + \nu^i_2 \bar{D}_t + \eta^i \xi_t\right], \tag{4.9}$$

where $\bar{D}_t$ is the average visible order imbalance over the past τ^i periods. The parameter $\nu^i_1 > 0$ determines a fundamentalist component in the trader's strategy, $\nu^i_2 > 0$ determines the impact of order imbalance on his expectations, and $\eta^i > 0$ – the impact of a noise component. These weights are drawn for the entire population of traders from three independent exponential distributions respectively:

$$\begin{aligned} f(\nu^i_1) &= \frac{1}{\sigma_{\nu_1}} e^{-\nu^i_1/\sigma_{\nu_1}}, && (4.10)\\ f(\nu^i_2) &= \frac{1}{\sigma_{\nu_2}} e^{-\nu^i_2/\sigma_{\nu_2}}, && (4.11)\\ \text{and} \quad f(\eta^i) &= \frac{1}{\sigma_{\eta}} e^{-\eta^i/\sigma_{\eta}}. && (4.12) \end{aligned}$$

The trading horizon of each trader is correlated with fundamentalist and imbalance components of his expectations and remains fixed over the course of trading in this market:[16]

$$\tau^i = \tau \left\lceil \frac{1 + \nu^i_1}{1 + \nu^i_2} \right\rceil. \tag{4.13}$$

Each trader observes transaction price time series p_t; if no transaction has occurred in the previous time stamp, then p_t is the mid-point price calculated as the average of current bid and ask prices. The forecasted rate of return $\hat{r}^i_{t,t+\tau^i}$ yields the maximum expected price

$$\hat{p}^i_{t+\tau^i} = p_t \exp(\hat{r}^i_{t,t+\tau^i}\tau^i). \tag{4.14}$$

In general, as can be deduced from (4.9), both buyers and sellers become more optimistic and raise the highest expected price p_M should the depth on the buy side exceed the depth on the sell side of the market. Similarly, if the supply given by the volume on the sell side Q^a_t is larger than the demand for the asset given by the volume on the buy side Q^b_t then incoming trader is more pessimistic and anticipates lower returns. Fundamentalist component, naturally, pulls the expected price up when the market is trading below the long-run level, and vice versa.

The portfolio of each trader is composed of stocks and cash $W^i_t = S^i_t p_t + C^i_t$ and his preferences

[16] For any $z \in \mathbb{R}$, the ceiling function $f(z) = \lceil z \rceil$ is the smallest integer greater or equal to z.

are described by a negative exponential (CARA) utility function:

$$U\left(W_t^i, \varphi^i\right) = -\exp(-\varphi^i W_t^i), \tag{4.15}$$

where $\varphi^i = \varphi \frac{1+\nu_1^i}{1+\nu_2^i}$ is a relative risk aversion coefficient of agent i proportional to his trading horizon and φ is a common risk aversion factor for the population of agents in this market. Each agent is initially assigned a random amount of stocks uniformly distributed on the interval $S_0 \in [0, N_S]$ and the amount of cash $C_0 \in [0, W_0]$, where his total wealth equals $W_0 = N_S p_0$. The initial allocation of wealth does not affect the outcome under CARA preferences. The optimal portfolio of each trader at any given point in time is determined by utility maximisation principle. I derive the demand for stocks of agent i in the following proposition.

Proposition 4.5.1. *Assume that a trader with a risk aversion φ^i and a portfolio of stocks and cash maximises the expected utility of his future wealth $W_{t+\tau^i}^i$. Let the trader form his expectation of the stock price $\hat{p}_{t+\tau^i}^i$ in τ^i periods according to rules (4.2) and (4.3). If the current price of the asset is p, then the trader's demand for a risky asset or the optimal number of stocks required to hold in his portfolio at time t is given by:*

$$\pi_t^i(p) = \frac{\ln(\hat{p}_{t+\tau^i}^i / p)}{\varphi^i V_t^i p}, \tag{4.16}$$

where V_t^i is the variance of the spot returns $V_t^i = \frac{1}{\tau^i} \sum_{j=1}^{\tau^i} (r_{t-j} - \bar{r}_t^i)^2$ with the mean spot return $\bar{r}_t^i = \frac{1}{\tau^i} \sum_{j=1}^{\tau^i} r_{t-j} = \frac{1}{\tau^i} \sum_{j=1}^{\tau^i} \ln\left(\frac{p_{t-j}}{p_{t-j-1}}\right)$.

Proof. Recall first that the utility function of the agent i at time t

$$U(W_t^i, \varphi^i) = -\exp(-\varphi^i W_t^i), \tag{4.17}$$

therefore, in τ^i periods ahead he expects

$$E_t\left[U(W_{t+\tau^i}^i, \varphi^i)\right] = E_t\left[-\exp(-\varphi^i W_{t+\tau^i}^i)\right], \tag{4.18}$$

where the future wealth is composed of $W_{t+\tau^i}^i = W_t^i + S_t^i (p_{t+\tau^i} - p_t)$. Assume that the asset returns are normally distributed, then the cumulative price change is approximated by $p_{t+\tau^i} - p_t = p_t(p_{t+\tau^i}/p_t - 1) \approx p_t \ln(p_{t+\tau^i}/p_t) = p_t r_{t+\tau^i}$, where $r_{t+\tau^i}$ is the log-return, therefore the wealth is

$$W_{t+\tau^i}^i = W_t^i + S_t^i p_t r_{t+\tau^i}. \tag{4.19}$$

Substituting this expression into (4.18) I obtain

$$\begin{aligned} E_t\left[U(W^i_{t+\tau^i},\varphi^i)\right] &= -E_t\left[\exp(-\varphi^i W^i_t - \varphi^i S^{\,i}_t p_t r_{t+\tau^i})\right] \\ &= -\exp(-\varphi^i W^i_t)\cdot E_t\left[\exp(-\varphi^i S^{\,i}_t p_t r_{t+\tau^i})\right] \\ &= U(W^i_t,\varphi^i)\cdot E_t\left[\exp(-\varphi^i S^{\,i}_t p_t r_{t+\tau^i})\right]. \end{aligned} \tag{4.20}$$

The only stochastic term unknown at time t in the equation (4.20) is the log-return $r_{t+\tau^i}$. The mean value of the return in τ^i periods is the expected return defined for each trader i in (4.2) and the measure of variance is the historical volatility over the past τ^i trading rounds, denoted V^i_t. Using the moment generating function property of a Gaussian random variable I rewrite

$$\begin{aligned} E_t\left[U(W^i_{t+\tau^i},\varphi^i)\right] &= U(W^i_t,\varphi^i)\cdot\exp\left(-\varphi^i S^{\,i}_t p_t \hat{r}^i_{t,t+\tau^i} + (\varphi^i S^{\,i}_t p_t)^2 V^i_t/2\right) \\ &= U(W^i_t,\varphi^i)\cdot\exp\left(-\varphi^i S^{\,i}_t p_t \ln(\hat{p}^i_{t+\tau^i}/p_t) + (\varphi^i S^{\,i}_t p_t)^2 V^i_t/2\right) \end{aligned} \tag{4.21}$$

The derivative of the expected utility with respect to the number of stocks $S^{\,i}_t$ is

$$\frac{\partial E_t[U]}{\partial S^{\,i}_t} = E_t[U]\cdot\left(-\varphi^i p_t \ln(\hat{p}^i_{t+\tau^i}/p_t) + (\varphi^i p_t)^2 V^i_t S^{\,i}_t\right). \tag{4.22}$$

Applying the first order condition $S^{\,i*}_t \partial E_t[U]/\partial S^{\,i*}_t = 0$ with $S^{\,i*}_t > 0$, I find a solution to the utility maximisation problem $\pi^i_t(p_t) = S^{\,i*}_t$ and arrive at expression (4.16). □

Denote the quantity that the trader is willing to purchase or to sell by $q^i_t(p) = \left|S^{\,i}_t - \pi^i_t(p)\right|$. For a price level p trader i wishes to buy more stock if his demand exceeds present stock holdings $\pi^i_t(p) > S^{\,i}_t$, while if his portfolio contains more stocks than required $\pi^i_t(p) < S^{\,i}_t$, then he wishes to reduce his exposure and sell stocks. There is a price p^*, a *satisfaction level*, that makes the current portfolio composition optimal. As a result, if the realised price is lower $p < p^*$ then the trader wishes to buy the security; conversely, he will be willing to sell when $p > p^*$. The range of admissible prices that a particular trader can choose is $p \in [p_m, p_M]$: the upper bound p_M is determined by a no short selling restriction and the lower bound p_m is subject to a no borrowing condition. Departing from the expected return in (4.2) and the associated expected price in (4.3), the trader i that is called to the market at time t solves the following three equations:

$$\forall p \le p_M : \ \pi^i_t(p) \ge 0 \quad \Leftrightarrow \quad p_M = \hat{p}^i_{t+\tau^i}, \tag{4.23}$$

$$\pi^i_t(p^*) = S^{\,i}_t \quad \Leftrightarrow \quad \frac{\ln(\hat{p}^i_{t+\tau^i}/p^*)}{\varphi^i V^i_t p^*} = S^{\,i}_t, \tag{4.24}$$

$$p_m\left(\pi_t^i(p_m) - S_t^i\right) = C_t^i \quad \Leftrightarrow \quad \frac{\ln(\hat{p}_{t+\tau^i}^i/p_m)}{\varphi^i V_t^i} - p_m S_t^i = C_t^i. \tag{4.25}$$

The trader is boundedly rational: he picks randomly a price p from a range of admissible prices in period t. A qualitative chart of the demand function $\pi_t^i(p)$ provided by Chiarella et al. (2009) is depicted in Figure 4.4. The relation of the boundaries p_m and p_M and the satisfaction price p^* to the current quotes, the bid b_t and ask a_t, determines the order aggressiveness of trader i at time t, as shown in Table 4.11.

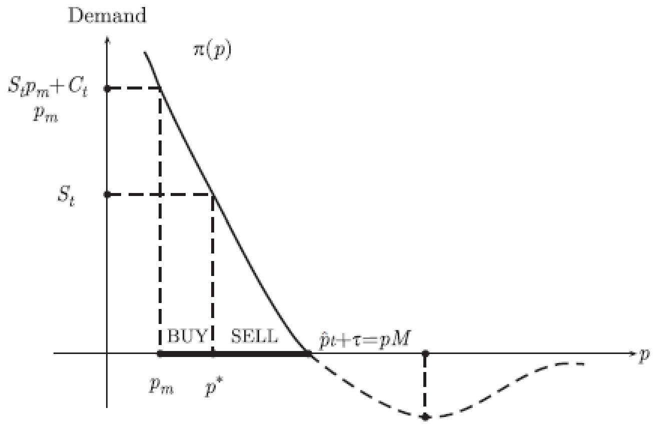


Figure 4.4: The demand for stock as a function of price and the corresponding range of admissible prices. Source: Chiarella et al. (2009).

Table 4.11: Order placement definitions for a chosen price p.

position	direction	type	volume
$p_m \leq p < a_t$	buy	limit	$q_t^i(p)$
$a_t \leq p < p^*$	buy	market	$q_t^i(a_t)$
$p = p^*$	no order	placement	
$p^* < p \leq b_t$	sell	market	$q_t^i(b_t)$
$b_t < p \leq p_M$	sell	limit	$q_t^i(p)$

Appendix B Proof of Proposition 4.2.1

Since neither of the equations (4.24) or (4.25) admits an explicit solution, I apply the implicit function theorem to obtain the derivatives of p^* and p_m with respect to p_M.

Theorem 4.5.1 (Implicit Function Theorem). *Let the function $F : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ have a zero in (x_0, y_0). If $F(x, y)$ is continuous in the vicinity of (x_0, y_0) and strictly monotonic in y for any fixed x,* *then there exists a continuous two-dimensional interval* $\mathbb{I} = \mathbb{I}_x \times \mathbb{I}_y$ *in the vicinity of* (x_0, y_0) *and a continuous function* $f : \mathbb{I}_x \to \mathbb{I}_y$ *such that for any point* $(x, y) \in \mathbb{I}$ *it holds:*

$$F(x, y) = 0 \quad \Leftrightarrow \quad y = f(x). \tag{4.26}$$

Corollary 4.5.1. *Suppose that* $F(x, y)$ *is continuously differentiable, and denote its partial derivative with respect to x by* F'_x *and the partial derivative with respect to y by* F'_y*. The partial derivative of the function* $f(x)$ *is expressed as*

$$f'(x) = -\frac{F'_x(x, f(x))}{F'_y(x, f(x))}. \tag{4.27}$$

First, to calculate $\partial p^*/\partial p_M$ define the function

$$F(p_M, p^*) = \frac{\ln(p_M/p^*)}{\varphi^i V_t^i p^*}, \tag{4.28}$$

then it follows from Corollary 4.5.1 that

$$\frac{\partial p^*}{\partial p_M} = -\frac{\partial F(p_M, p^*)/\partial p_M}{\partial F(p_M, p^*)/\partial p^*} = \frac{(p^*)^2 \varphi^i V_t^i}{p_M p^* \varphi^i V_t^i (1 + p^* \varphi^i V_t^i S_t^i)}.$$

Next, I apply substitution $\ln(p_M/p^*) = p^* \varphi^i V_t^i S_t^i$ implied by (4.24) and obtain

$$\frac{\partial p^*}{\partial p_M} = \frac{p^*}{p_M(1 + p^* \varphi^i V_t^i S_t^i)}. \tag{4.29}$$

Second, to calculate $\partial p_m/\partial p_M$ define the function

$$G(p_M, p_m) = \frac{\ln(p_M/p_m)}{\varphi^i V_t^i} - p_m S_t^i, \tag{4.30}$$

then by the property of implicit function (4.27)

$$\frac{\partial p_m}{\partial p_M} = -\frac{\partial G(p_M, p_m)/\partial p_M}{\partial G(p_M, p_m)/\partial p_m} = \frac{p_m}{p_M(1 + p_m \varphi^i V_t^i S_t^i)}. \tag{4.31}$$

Notice, that derivatives (4.29) and (4.31) are both positive since all the parameters are positive and have the same functional form $f(x) = x/(1 + x\varphi^i V_t^i S_t^i)$. Given that function $f(x)$ is strictly monotonic increasing ($f'(x) > 0 \ \forall x \in \mathbb{R}$), it follows

$$\frac{\partial p^*}{\partial p_M} > \frac{\partial p_m}{\partial p_M} > 0. \tag{4.32}$$

Appendix C Proof of Proposition 4.2.2

The trader picks any price $p \in [p_m, p_M]$ with an equal probability, therefore, in order to evaluate the impact of a certain factor z on the propensity of trader i who arrived to the market at time t to buy or sell shares, it suffices to compute the derivative of the ratio of the appropriate subinterval (buy or sell) to the interval of admissible prices with respect to z. Consider the sell subinterval $[p^*, p_M]$ and denote its ratio to the full price range by

$$A = \frac{p_M - p^*}{p_M - p_m}, \tag{4.33}$$

then

$$\frac{\partial A}{\partial z} = \frac{(p_M - p_m)\left(\frac{\partial p_M}{\partial z} - \frac{\partial p^*}{\partial z}\right) - (p_M - p^*)\left(\frac{\partial p_M}{\partial z} - \frac{\partial p_m}{\partial z}\right)}{(p_M - p_m)^2}. \tag{4.34}$$

If $\partial A/\partial z > 0$, then the trader becomes more likely to be a seller once z increases; if $\partial A/\partial z < 0$, then the trader becomes more likely to be a buyer once z increases. I first establish the necessary results for statements (1), (2a) and (2b), and then prove the effect of volatility in statement (3).

Statements (1), (2a) and (2b)

Since p^* and p_m depend on the order imbalance $\bar{D}_t$ and v_2^i only through p_M, to prove statements (1), (2a) and (2b) of Proposition 4.2.2 I write the differentials of prices with respect to a change in factor z as follows:

$$\frac{\partial p^*}{\partial z} = \frac{\partial p^*}{\partial p_M} \cdot \frac{\partial p_M}{\partial z}, \quad \text{and} \quad \frac{\partial p_m}{\partial z} = \frac{\partial p_m}{\partial p_M} \cdot \frac{\partial p_M}{\partial z},$$

therefore, in this case expression (4.34) simplifies to

$$\frac{\partial A}{\partial z} = \frac{\partial p_M/\partial z}{(p_M - p_m)^2}\left[(p_M - p_m)\left(1 - \frac{\partial p^*}{\partial p_M}\right) - (p_M - p^*)\left(1 - \frac{\partial p_m}{\partial p_M}\right)\right]. \tag{4.35}$$

By substituting the expressions (4.29) and (4.31) from Appendix B, I rewrite the derivative of term A as follows:

$$\frac{\partial A}{\partial z} = \frac{\partial p_M/\partial z}{p_M(p_M - p_m)^2}\underbrace{\left[(p_M - p_m)\left(p_M - \frac{p^*}{1 + p^*\varphi^i V_t^i S_t^i}\right) - (p_M - p^*)\left(p_M - \frac{p_m}{1 + p_m\varphi^i V_t^i S_t^i}\right)\right]}_{B_1}.$$

Given that by definition $p_m < p^* < p_M$, substitute $p^* = a \cdot p_m$ and $p_M = b \cdot p_m$, where $b > a > 1$.

Then term B_1 is expressed by

$$\begin{aligned} B_1 &= p_m(b-1)\left(bp_m - \frac{ap_m}{1+ap_m\varphi^i V_t^i S_t^i}\right) - p_m(b-a)\left(bp_m - \frac{p_m}{1+p_m\varphi^i V_t^i S_t^i}\right) \\ &= p_m^2(ab + abp_m\varphi^i V_t^i S_t^i + b - a)\left[\frac{1}{1+p_m\varphi^i V_t^i S_t^i} - \frac{1}{1+ap_m\varphi^i V_t^i S_t^i}\right]. \end{aligned}$$

Notice that function $f(x) = 1/(1 + x\varphi^i V_t^i S_t^i)$ is monotonic increasing $\forall x \in \mathbb{R}$, hence $B_1 > 0$ and the sign of $\partial p_M/\partial z$ determines the change in the propensity to sell. Denote the weight of order imbalance in the expected return by $\tilde{\nu}_2^i = \nu_2^i/(\nu_1^i + \nu_2^i + \eta^i)$. Recall that from equation (4.3) the maximum price is $p_M = p_t \exp(\hat{r}^i_{t,t+\tau^i}\tau^i)$, so the derivatives with respect to the visible imbalance $\bar{D}_t$ and imbalance weight $\tilde{\nu}_2^i$ determine the sign of $\partial A/\partial \bar{D}_t$ and $\partial A/\partial \tilde{\nu}_2^i$ as follows

$$\frac{\partial p_M}{\partial \bar{D}_t} = p_M \tau^i \tilde{\nu}_2^i, \quad \Rightarrow \quad \frac{\partial A}{\partial \bar{D}_t} > 0, \tag{4.36}$$

$$\text{and} \quad \frac{\partial p_M}{\partial \tilde{\nu}_2^i} = p_M \tau^i \bar{D}_t, \quad \Rightarrow \quad \operatorname{sgn}\left\{\frac{\partial A}{\partial \tilde{\nu}_2^i}\right\} = \operatorname{sgn}\{\bar{D}_t\}. \tag{4.37}$$

Therefore, I conclude that the propensity to sell increases when the order imbalance in the limit order book increases towards buyers. Similarly, higher sensitivity to imbalance prompts the trader to join the opposite side of the market relative to the visible imbalance trend.

Statement (3)

Statement (3) of Proposition 4.2.2 is proven in a similar manner. Notice, that the maximum expected price p_M is not related to asset return volatility in this model, therefore, $\partial p_M/\partial V_t^i = 0$. The minimum and satisfaction prices depend on the spot volatility, so I simplify the expression of the derivate of A with respect to V_t^i as follows

$$\frac{\partial A}{\partial V_t^i} = \frac{(p_M - p^*)\frac{\partial p_m}{\partial V_t^i} - (p_M - p_m)\frac{\partial p^*}{\partial V_t^i}}{(p_M - p_m)^2}. \tag{4.38}$$

I define two functions

$$\tilde{F}\left(V_t^i, p^*\right) = \frac{\ln(p_M/p^*)}{\varphi^i V_t^i p^*}, \quad \text{and} \quad \tilde{G}\left(V_t^i, p_m\right) = \frac{\ln(p_M/p_m)}{\varphi^i V_t^i} - p_m S_t^i, \tag{4.39}$$

and applying the corollary of implicit function theorem (4.27), I compute partial derivatives with respect to the spot volatility V_t^i:

$$\frac{\partial p^*}{\partial V_t^i} = -\frac{\partial \tilde{F}\left(V_t^i, p^*\right)/\partial V_t^i}{\partial \tilde{F}\left(V_t^i, p^*\right)/\partial p^*} = -\frac{p^* \ln(p_M/p^*)}{V_t^i(1 + p^*\varphi^i V_t^i S_t^i)}, \tag{4.40}$$

$$\text{and} \quad \frac{\partial p_m}{\partial V_t^i} = -\frac{\partial \tilde{G}\left(V_t^i, p_m\right)/\partial V_t^i}{\partial \tilde{G}\left(V_t^i, p_m\right)/\partial p_m} = -\frac{\ln(p_M/p_m)}{V_t^i(1 + p_m\varphi^i V_t^i S_t^i)}. \tag{4.41}$$

I substitute the derivatives (4.40) and (4.41) into the expression (4.34) and obtain

$$\frac{\partial A}{\partial V_t^i} = \frac{1}{V_t^i(p_M - p_m)^2} \underbrace{\left[(p_M - p_m)\frac{p^* \ln(p_M/p^*)}{(1 + p^*\varphi^i V_t^i S_t^i)} - (p_M - p^*)\frac{\ln(p_M/p_m)}{(1 + p_m\varphi^i V_t^i S_t^i)}\right]}_{B_2}.$$

Further, I apply the same substitution $p^* = a \cdot p_m$ and $p_M = b \cdot p_m$ with $b > a > 1$, and rearrange term B_2 as follows

$$\begin{aligned} B_2 &= p_m(b-1)\frac{ap_m \ln(b/a)}{(1 + ap_m\varphi^i V_t^i S_t^i)} - p_m(b-a)\frac{\ln(b)}{(1 + p_m\varphi^i V_t^i S_t^i)} \\ &= (b-1)\ln(b/a)\left[p_m \frac{ap_m}{(1 + ap_m\varphi^i V_t^i S_t^i)} - \frac{\ln(b)}{b-1} \cdot \frac{b-a}{\ln(b/a)} \cdot \frac{p_m}{(1 + p_m\varphi^i V_t^i S_t^i)}\right]. \end{aligned}$$

Notice that function $f(x) = 1/(1 + x\varphi^i V_t^i S_t^i)$ is monotonic increasing in x, therefore,

$$\frac{ap_m}{(1 + ap_m\varphi^i V_t^i S_t^i)} > \frac{p_m}{(1 + p_m\varphi^i V_t^i S_t^i)}. \tag{4.42}$$

On the other hand, function $g(x) = \ln(b/x)/(b - x)$ is monotonic decreasing in x, so

$$\frac{\ln(b)}{b-1} \cdot \frac{b-a}{\ln(b/a)} > 1.$$

Taking into account $\lim_{a \to b^-}\left[\frac{\ln(b)}{b-1} \cdot \frac{b-a}{\ln(b/a)}\right] = \frac{\ln(b)}{b-1} \cdot \lim_{a \to b^-}\left[\frac{\partial(b-a)/\partial a}{\partial \ln(b/a)/\partial a}\right] = \frac{b \cdot \ln(b)}{b-1}$, it follows that[17]

$$p_m > \frac{\ln(b)}{b-1} \cdot \frac{b-a}{\ln(b/a)}. \tag{4.43}$$

The product of the two inequalities (4.42) and (4.43) thus yields $B_2 > 0$ and

$$\frac{\partial A}{\partial V_t^i} > 0, \tag{4.44}$$

[17]This inequality holds for almost all price levels, unless both p^* and p_m are near zero while p_M is substantially high. The range of market parameters that I use in my simulations, in particular, common risk aversion φ and the maximum initial number of stocks per trader N_A, prevent such situations.

which proves statement (3) of Proposition 4.2.2. Therefore, I establish that the trader's propensity to sell increases with the volatility of returns in this model.

Appendix D Nonparametric Test for the Equality of Distributions

I use the basic methodology described in Li and Racine (2007, p.465). Given $\{X_i\}_{i=1}^{N_1}$ and $\{Y_i\}_{i=1}^{N_2}$ are two unidimensional random samples, I test if they are drawn from the same distribution using nonparametric kernel density estimators. The integrated squared difference between two empirical distributions is given by

$$I_n^b = \frac{1}{N_1}\sum_{i,j}^{N_1} K_{h,ij}^x + \frac{1}{N_2}\sum_{i,j}^{N_2} K_{h,ij}^y - \frac{2}{N_1 N_2}\sum_{i}^{N_1}\sum_{j}^{N_2} K_{h,ij}^{xy}, \tag{4.45}$$

where kernel functions are defined through a bounded second order kernel $k(\cdot) \geq 0$ and a bin width h as

$$K_{h,ij}^x = \frac{1}{h}k\left(\frac{X_i - X_j}{h}\right),\ K_{h,ij}^y = \frac{1}{h}k\left(\frac{Y_i - Y_j}{h}\right),\ and\ K_{h,ij}^{xy} = \frac{1}{h}k\left(\frac{X_i - Y_j}{h}\right).$$

The null hypothesis of the test H_0^b assumes that the distribution of $\{X_i\}_{i=1}^{N_1}$ and $\{Y_i\}_{i=1}^{N_2}$ are equal for almost all x. The test statistic then follows a normal distribution and is calculated as

$$T_n^b = \sqrt{hN_1N_2}\frac{I_n^b - c_{n,b}}{\hat{\sigma}_b} \sim N(0,1) \tag{4.46}$$

with $c_{n,b} = \frac{k(0)(N_1+N_2)}{hN_1N_2}$ and the sample variance is obtained as

$$\hat{\sigma}_b^2 = \frac{1}{N_1^2}\sum_{i,j}^{N_1} h\cdot(K_{h,ij}^x)^2 + \frac{1}{N_2^2}\sum_{i,j}^{N_2} h\cdot(K_{h,ij}^y)^2 + \frac{2}{N_1N_2}\sum_{i}^{N_1}\sum_{j}^{N_2} h\cdot(K_{h,ij}^{xy})^2. \tag{4.47}$$

The equality hypothesis H_0^b is rejected at level α if $T_n^b > z_\alpha$, where z_α is the α% percentile from the standard Normal distribution.

Appendix E Empirical Order Flow Data

In the original paper, Biais et al. (1995) distinguish between 15 order types, which I regroup for the purpose of comparison with my results. Since there are no strategic order cancellations in my artificial market, I disregard order types 'Cancel Ask' and 'Cancel Bid' as well as pre-arranged trades labelled as 'Application' that are absent in my market. I combine 'Large Buy', 'Market Buy' and 'Small Buy' into buy market orders (BM) since these orders result in immediate execution, 'New Bid Within' as an aggressive buy limit order (aBL), 'New Bid At' and 'New Bid Below' ito passive buy limit orders (pBL). Similarly, 'Large Sell', 'Market Sell' and 'Small Sell' are treated as sell market orders (SM), 'New Ask Within' as an aggressive sell limit order (aSL), and, finally, 'New Ask At' and 'New Ask Above' as passive sell limit orders (pSL).

Table 4.12: Percentage deviations of order type probabilities conditional on a previously submitted order type from their unconditional probabilities that are given in the last column.

Paris Bourse

	BM	aBL	pBL	SM	aSL	pSL	Unconditional
BM	**57.5**	3.6	-3.7	-20.2	-31.0	-8.8	19.03
aBL	9.8	**52.7**	25.4	-22.1	0.1	-27.2	10.10
pBL	-3.6	23.9	**62.4**	-17.5	-22.2	-22.6	14.52
SM	-19.0	-24.7	-20.0	**33.3**	-4.1	-17.2	34.23
aSL	-18.7	5.1	-21.9	-2.3	**51.5**	15.4	9.37
pSL	-11.6	-24.1	-22.5	-12.2	9.1	**85.1**	12.96

In a similar manner I transform the data from Hall and Hautsch (2006) by combining 'Aggressive Buy/Sell' and 'Normal Buy/Sell' into buy/sell market orders (BM/SM), 'Aggressive Bid/Ask' and 'Normal Bid/Ask' into passive buy/sell limit orders (pBL/pSL) and write 'Most Aggressive Bid/Ask' as aggressive buy/sell limit orders (aBL/aSL). The results for the five stocks quoted on the Australian Stock Exchange are given in Table 4.13. The stocks abbreviations stand for Broken Hill Proprietary Limited (BHP), National Australia Bank (NAB), News Corporation (NCP), Telstra (TLS) and Woolworths (WOW).

Table 4.13: The unconditional probabilities of six order types (in %) in the Australian Stock Exchange.

Australian Stock Exchange

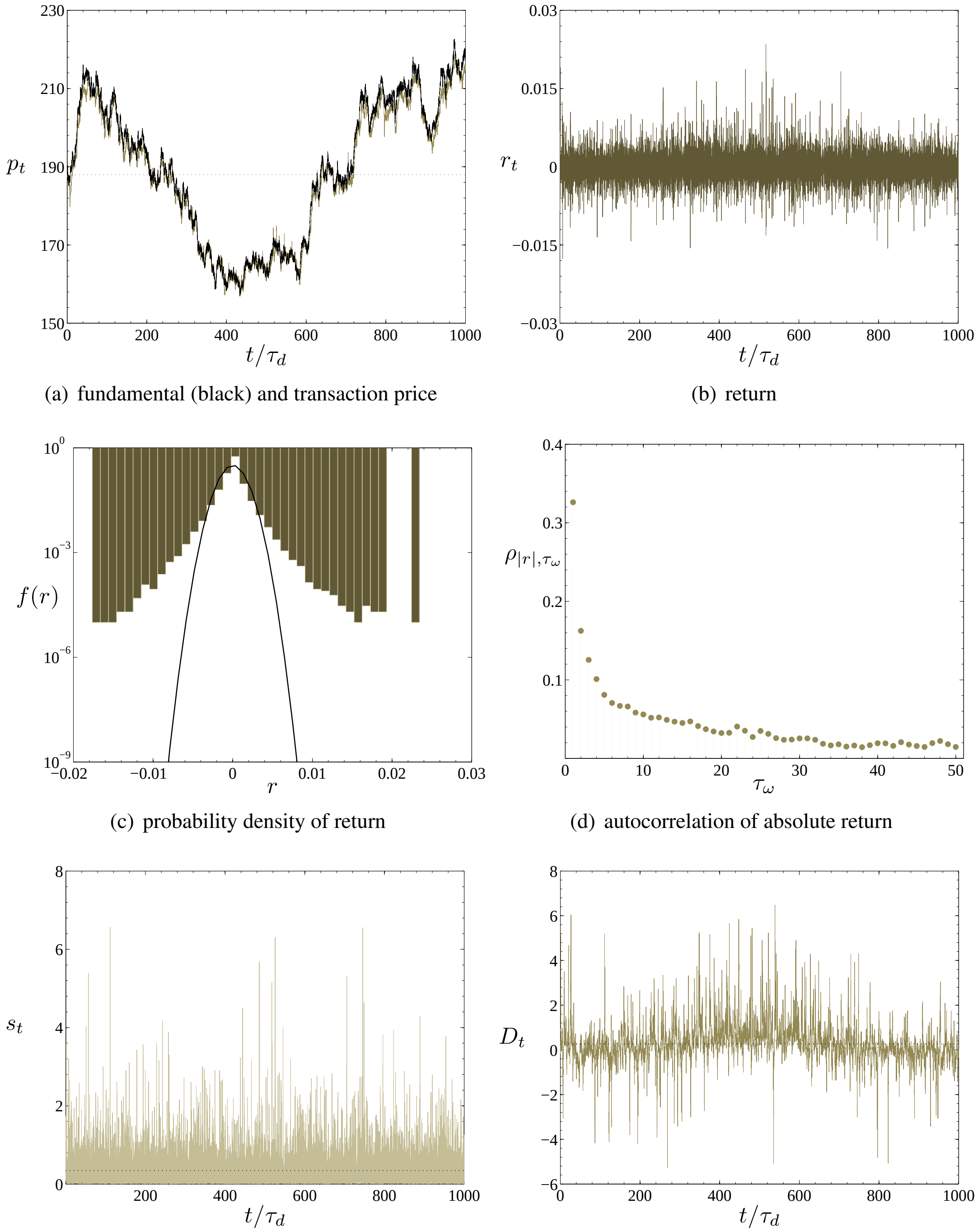
	BHP	NAB	NCP	TLS	WOW
BM	27.72	24.38	15.91	23.99	25.43
aBL	5.64	5.93	3.58	1.79	4.36
pBL	24.70	20.93	29.17	25.72	24.16
SM	18.28	23.37	13.81	22.40	20.92
aSL	4.28	6.29	4.52	1.29	5.78
pSL	19.38	19.10	33.01	24.81	19.34

Appendix F Statistics of a Single Simulation Run

This section provides summary statistics and profile plots of one simulation run with 100,000 rounds under four transparency regimes.

Table 4.14: Summary statistics of one simulation run under four transparency regimes.

	Dark	*Transparent*	*Quasi-Transparent*	*Opaque*
Transaction Price				
mean	187.96	186.18	184.70	188.42
s.d.	17.14	15.81	16.24	15.23
Returns				
min	-0.0176	-0.1061	-0.0473	-0.0529
max	0.0235	0.0626	0.0755	0.0306
mean	0.0000	0.0000	0.0000	0.0000
s.d.	0.0013	0.0024	0.0026	0.0014
skewness	0.3115	-0.4307	0.5503	-1.2746
kurtosis	18.2475	69.2100	35.6471	73.4915
Spread				
mean	0.34	0.64	0.67	0.44
s.d.	0.34	0.70	0.77	0.48
Visible Imbalance				
mean	0.0000	0.2674	0.0890	0.1134
s.d.	0.0000	0.7830	0.8831	0.5526
Total Imbalance				
mean	0.2651	0.2674	0.3718	0.1744
s.d.	0.7234	0.7830	0.8872	0.7728
Visible Book Depth				
average buy depth	0.00	704.22	71.52	276.67
average sell depth	0.00	490.2765	74.30	224.18
average book depth	0.00	1,194.50	145.81	500.85
Total Book Depth				
average buy depth	524.18	704.22	689.51	602.67
average sell depth	391.77	490.2765	459.54	482.61
average book depth	915.94	1,194.50	1,149.05	1,085.28
Order Flow				
no. of orders	97,075	97,030	97,074	70,995
average order size	23.93	23.60	23.66	29.78
average BM	29.38	27.80	27.63	36.81
average BL	21.67	22.46	22.49	25.76
average SM	32.15	30.85	30.69	39.62
average SL	19.16	19.07	19.16	23.60
Transactions				
no. of trades	74,754	71,647	72,049	78,345
trading volume	1,013,767	939,414	954,464	1,016,113
average trade size	13.56	13.11	13.25	12.97



(a) fundamental (black) and transaction price

(b) return

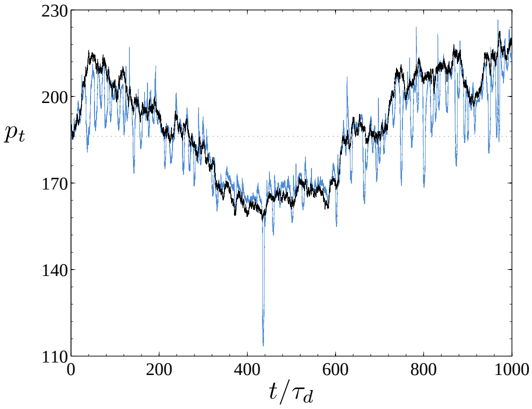
(c) probability density of return

(d) autocorrelation of absolute return

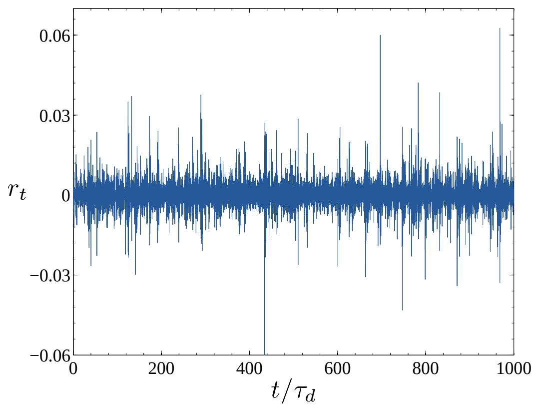
(e) bid-ask spread

(f) order imbalance

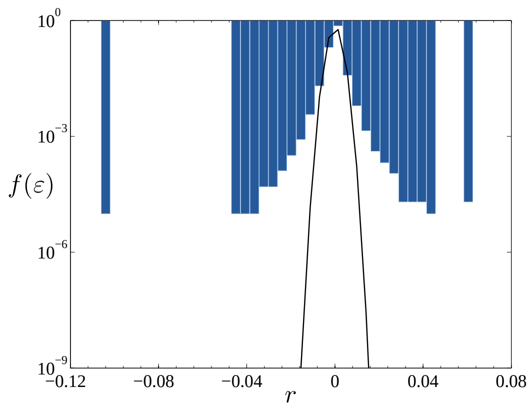
Figure 4.5: Profile plots of data series in the dark market with parameters in Table 4.2.



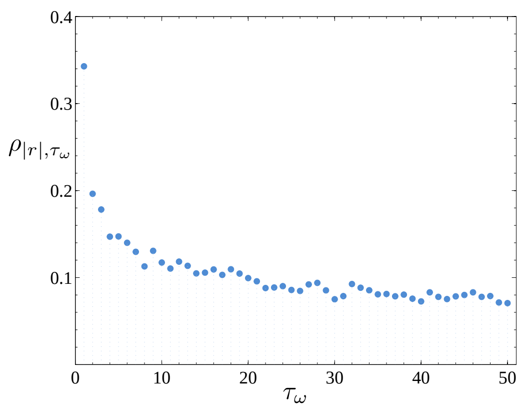
(a) fundamental (black) and transaction price



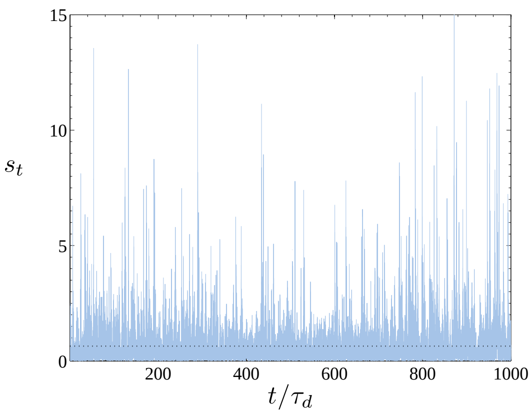
(b) return



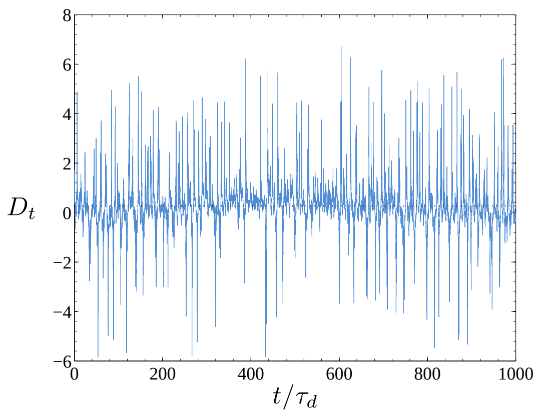
(c) probability density of return



(d) autocorrelation of absolute return

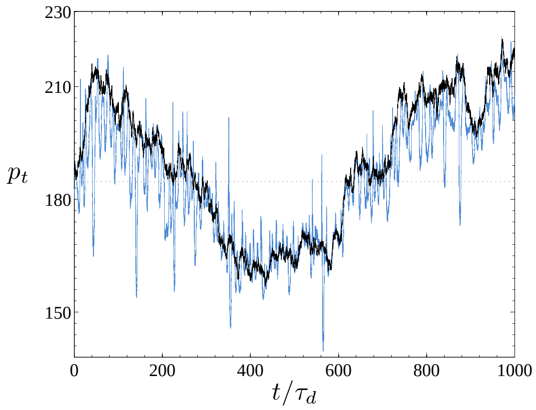


(e) bid-ask spread

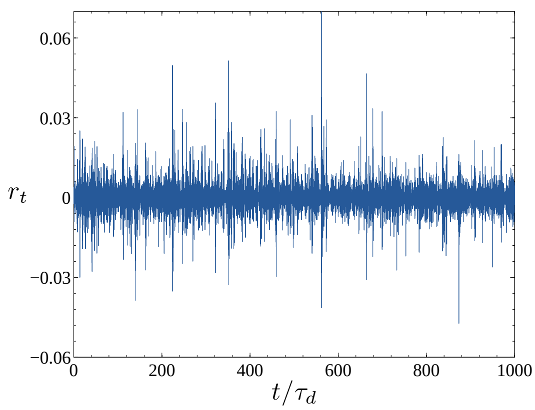


(f) order imbalance

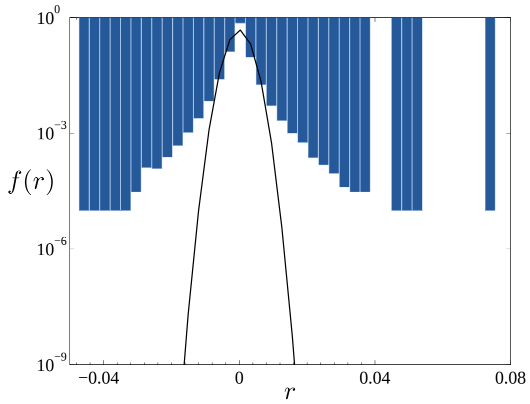
Figure 4.6: Profile plots of data series in the transparent market with parameters in Table 4.2.



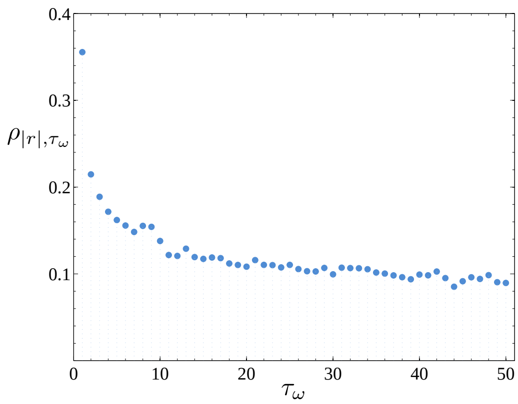
(a) fundamental (black) and transaction price



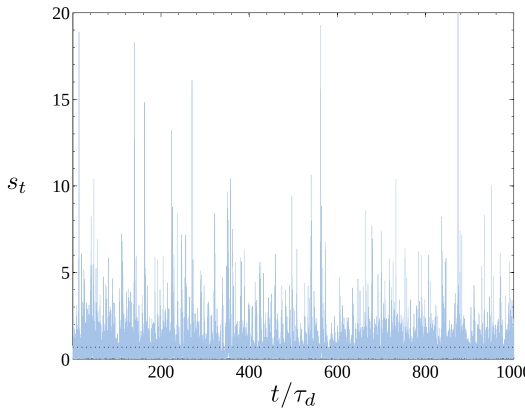
(b) return



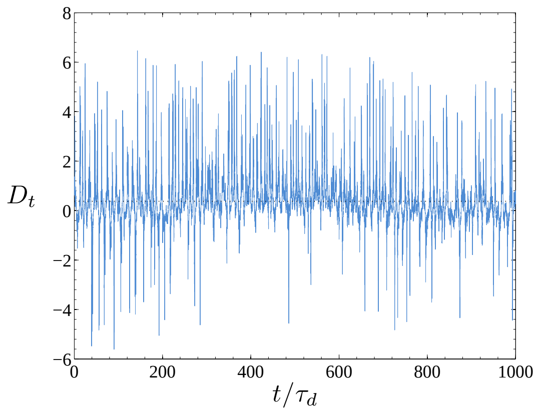
(c) probability density of return



(d) autocorrelation of absolute return



(e) bid-ask spread



(f) order imbalance

Figure 4.7: Profile plots of data series in the quasi-transparent market with parameters in Table 4.2.

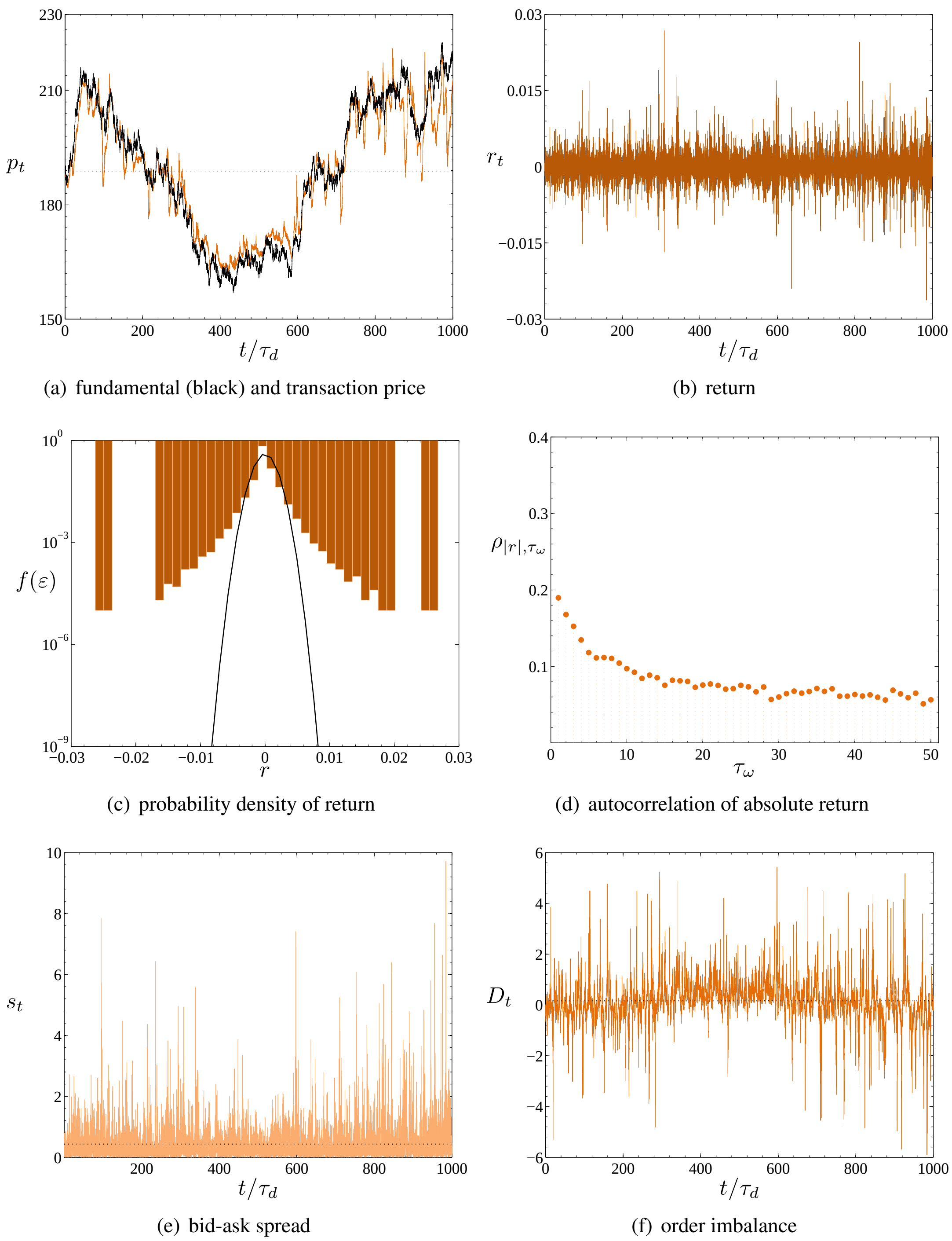


Figure 4.8: Profile plots of data series in the opaque market with parameters in Table 4.2.

Chapter 5

Conclusions

This thesis contributes to the theoretical domain of microstructure economics by exploring two features of financial markets: liquidity and market transparency. My research is motivated by the progressive expansion of electronic markets with reduced pre-trade transparency and the collateral liquidity effects. Maintaining sufficient liquidity is an endemic problem of any organised exchange, especially in the absence of designated specialists responsible for liquidity provision, as in the case of non-intermediated order-driven markets. Liquidity improvement is, therefore, a primary goal of regulators, policy-makers and market operators alike. Iceberg and hidden orders are designed to attract liquidity by protecting institutional block traders. However, there orders obfuscate quote transparency and, according to diverse empirical evidence, have controversial influence on market quality. Aiming to provide economic explanation to empirical facts, I developed three theoretical frameworks in this thesis to address the connection between liquidity and transparency and the implications of imperfect liquidity from various angles. I have investigated liquidity and transparency issues in the context order placement decisions made by individual traders. Further, I evaluated the interplay of transparency and liquidity determinants in the artificial agent-based stock exchange environment.

5.1 Main Findings

A substantial challenge of microstructure analysis is to disentangle the effects and infer causality between market transparency and liquidity, given the multi-dimensional nature of these concepts. As the core finding of this thesis I have identified the direction of mutual dependence between liquidity and transparency on the basis of several quantitative tools, and reconciled certain observed empirical phenomena.

Abstracting initially from transparency issues, I showed through an optimal limit order pricing model that departures from absolute liquidity serve as a source of non-transient bid-ask spread in pure limit order markets. Moreover, by introducing a synthetic liquidity factor, measured by a random delay in limit order execution, I demonstrated that improvement in liquidity reduces the equilibrium bid-ask spread while a higher traders' risk aversion widens it. My agent-based simulation analysis suggests the same tendency: the bid-ask spread is tighter in markets with subsistent liquidity.

Furthermore, my comparative analysis of four distinct transparency regimes in the artificial double auction market reveals that permission of iceberg orders alleviates the problem of adverse selection, improves market efficiency and liquidity properties, and reduces the bid-ask spread, which represents the transaction costs and is one of the key market quality factors. Nonetheless, my experiments indicate that full quote transparency incurs substantial transaction costs for traders and dampens trading activity in an order-driven market. At the same time, according to simulation results, exogenous restriction of market depth transparency to the five best quotes on each side of the book has no substantial effect on market performance, unlike self-regulation of market transparency via iceberg orders. Further scrupulous analysis in the real option setup delineates the situations when the intensity of the order flow can accentuate the willingness of the trader to participate in the opaque market. I showed that once the trader enters the opaque market, he commits to trade rather quickly relative to his time horizon. Therefore, I conclude that there is a positive relationship between market opacity and liquidity, and that permitting traders to disguise part of their orders is desirable for the mutual benefit of market participants.

Although opacity has advantages on the aggregate level, it entails downsides from the standpoint of individuals: due to the imperfect clarity of information signals embedded in the visible order flow, impatient traders are highly likely to surpass the favourable trade execution offered in the opaque market, while a prior optimistic bias prompts traders to place their orders sooner. In addition, my model of optimal order submission time suggests that opaque markets have a natural predisposition to attract speculators compared to transparent markets, which, in turn, can potentially hinder price discovery and deteriorate market quality.

This thesis, therefore, provides a theoretical interpretation of both the individual investors' attitude towards market opacity and imperfect liquidity, and the resulting emergent market properties. In line with numerous empirical observations, it demonstrates that opaque markets fulfil its mission to enhance aggregate market quality and augment liquidity with few downsides for individual traders.

5.2 Potential Extensions

The three original models developed in this thesis can be employed to advance the understanding of patterns evicted by concrete transparency policies and consequent liquidity supply in multiple directions.

The model of optimal limit order pricing in a market with imperfect liquidity has been analysed in the form of a a static solution. Allowing for revisions of limit order price over time, I intend to derive a dynamic optimal strategy. In this version the model can also be applied to examine the problem of a block trader with a random delay factor dependent on the total volume that the trader is willing to trade. These extensions would contribute to the branch of microstructure literature that focuses on optimal dynamic trading, in particular, splitting strategies. Furthermore, a different stochastic process can be used to describe the dynamics of best prices, such as a double-exponential diffusion, which incorporates jumps.

The model of optimal order submission time in the opaque market would benefit from accounting for the stochastic nature of asset prices. However, the model may not admit an analytical solution, and numerical methods would be required instead. Furthermore, the range of execution prices that the trader expects to achieve could be linked to the visible order imbalance in order to make the model more realistic. It would also be interesting to examine whether various heuristics, such as loss aversion and availability bias motivate even further or discourage traders to participate in opaque trading.

Lastly, in future work I could extend the simulation analysis to incorporate the adaptive behaviour of the heterogeneous agents that populate the artificial limit order market. The current version of the agent-based model could be adjusted to allow learning among all agents in order to explore their reaction to the change of transparency policy from full quote disclosure to opaque market and the reverse shock. This setup would enable to conduct a comparative study alike live experiments from real markets and juxtapose the effects of imposing lower transparency versus the effects of subsequent unveiling of the limit order book.

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